JOURNAL OF ECONOMIC THEORY 25, 165–195 (1981)

An Equilibrium Analysis of Search and Breach of Contract II. A Non-Steady State Example

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Received May 8, 1979; revised June 12, 1980

1. INTRODUCTION

Consider a market where individuals meet pairwise and where each individual makes at most one trade (or is part of at most one project). The market for waterfront summer rentals is one example. Assume that search is costly and that there are fixed (and equal) numbers of potential landlords and renters. Suppose that tastes differ in that two renters may disagree about which of two houses is better, but assume that houses are ex ante identical in the sense that the distribution of evaluations is the same for each. For simplicity, assume only two possible evaluations: good and poor. In this paper, we analyze the equilibrium path over time of search and trade in such a market. We are particularly interested in its deviations from efficiency.

When the market opens, potential renters search for houses to rent, and landlords seek potential renters. Searchers meet according to a Poisson process. When a meeting results in a good match (a meeting where the renter's evaluation of the house is "good"), a rental is negotiated, a lease is signed, and both parties stop searching. When a poor match occurs, the parties also negotiate and sign a contract if neither already has a partner. However, they both search for a better deal if the expected benefits of further search exceed the costs. If neither partner finds a better deal, the two will ultimately carry out the negotiated rental. If one is successful[1], however, he can break the original lease and compensate his former partner for the loss borne. This better match might be with an individual who has not previously signed a lease. In that case, breaking the original lease is called a *single breach*. Alternatively, the new partner may already be matched, albeit poorly. Then, forming a new arrangement involves breaking both the old leases, a *double breach*. The process terminates when no one wishes to

[1] In our model the probability of simultaneous meetings is zero, so we can ignore the possibility that they both find better matches at the same time.

0022-0531/81/050165-31$02.00/0

continue searching, at which point all contracts remaining in existence are carried out.

We consider two distinct meeting processes or search technologies. In the first, the probability of a given renter's meeting a given landlord (assuming they are both searching) is independent of the number of other searchers. This process is designated the *quadratic case* since the rate of meetings rises with the square of the number of searchers. In the second meeting process, the probability of an individual's meeting someone at all is independent of the number of searchers. This is the *linear* technology.

With the quadratic technology, an equilibrium path[2] can assume several different patterns. What follows is one possibility. From the start of the search process until some time t_1, everyone who has not yet found a good match continues searching, and individuals make double and single breaches whenever they form a good match. From t_1 to t_2, those without good matches continue to search but now make only single breaches, since double breach is no longer worthwhile. At t_2, search ceases to be profitable for anyone already matched, and so only the partnerless continue. Finally, at time t_3, even they no longer find search worthwhile, and the process terminates. This sequence exhibits four different modes of behavior or configurations: A, where all individuals without good matches search, making both single and double breaches; B, where again those without good matches search but only single breaches occur; C, where only the partnerless search; and D, where there is no search. Since the order is A, B, C, D, the example is an ABCD sequence. We show below that the only other sequences possible are ACD, AD, BCD, CD, and D. Which one in fact occurs depends on the parameters of the economy and the starting point.

Besides describing equilibrium, the paper considers two efficiency questions. The first concerns the efficiency implications of small perturbations in the times t_i where the equilibrium path changes configurations. Returning to the above example and assuming that behavior is not changed before t_1, an increase in t_1, corresponding to a postponement of the cessation of double breaches, increases aggregate net output. In the neighborhood of t_1, a double breach alters the search environment in a way that others find valuable. The external economy implies, therefore, that, in equilibrium, Configuration A ends prematurely. Efficiency is also improved by increasing t_2; that is, aggregate output is increased if the time when those with poor matches stop searching is delayed. By contrast, perturbing t_3, the time when the partnerless stop searching, cannot improve efficiency. Because everyone searching at t_3 is identical (i.e., partnerless), each finds search to have zero marginal value at exactly the time when the social value of search (the sum of the individual values) is also zero.

[2] By "equilibrium path" we mean the trajectory that the numbers of searchers follow when, at each instant, every searcher maximizes his expected net gain, given the behavior of others.

Our second approach to efficiency is analysis of the optimal path. Here we derive the first order conditions of the times when configurations change. One interesting conclusion we draw is that the optimal path never involves Configuration B behavior (search by everyone without a good match but no double breach).

These results apply to the quadratic technology. With the linear technology, the perceived environment never changes for a typical searcher, and, therefore, behavior is uniform throughout the process. The equilibrium, moreover, is efficient.

This paper is concerned with much the same issues as was our previous study [1]. The principal distinction between the papers is that formerly we postulated the continuous arrival of new searchers and thus analyzed steady state behavior, whereas now we are concerned with the evolution of the search process.

After setting up the model (Section 2), the paper begins with consideration of the quadratic process: Section 3 examines the equilibrium path assuming full compensation for breach of contract; Section 4, the efficiency implications of small changes from the equilibrium path; Section 5, the efficient path; and Section 6, the equilibrium path assuming no compensation for breach. The linear process is presented in Section 7. We conclude with a brief summary and some remarks on the generality of the results.

2. The Model[3]

We consider a model with two types of individuals.[4] Individuals are distinguished by type only in that each partnership (contract) requires exactly one partner of each type. Individuals search for a partner (of the opposite type) with whom to undertake a single project. If partners are well-matched, the project is worth $2X$. If not, output is $2X'$. We assume $X > X' > 0$. After partners have stopped searching—and only then—the project corresponding to their partnership is completed. Individuals are risk neutral and are able to make side payments with no bankruptcy constraints. Each individual can engage in at most one project and belong to at most one partnership.

Individuals can meet new potential partners only if they search, and the cost of search is a flow, c, per unit time. Under the quadratic technology, the probability that any two searchers (of opposite types) meet is a per unit time. Under the linear technology, a is the probability per unit time that a given searcher meets someone at all. We assume a is sufficiently small so

[3] The model described below is essentially that of our earlier paper [1] and, therefore, is not discussed so fully as before.

[4] For example, buyers and sellers or lessors and lessees.

that we can ignore the possibility that two partners who are both searching will simultaneously find new potential partners.[5] When two individuals meet, the probability of their matching poorly is p, with $1-p$, the probability they are a good match. All parameters are the same for individuals of both types, and so we shall refer to just one type.

Let a partnerless individual be designated by "M," and let "N" refer to an individual with a poor contract. h_M denotes the number of M's, and h_N, the number of N's. For most of our analysis, we can disregard the number of individuals with good contracts, since they never search. When the search process begins there are $h_M(0)$ M's and no N's.

As in the Introduction, we classify search and breach behavior among four configurations. As we shall see, if two M's meet it will be in their interest to sign a contract regardless of the quality of match. However, N's, will breach only to form good contracts. Our interest in breaching behavior centers on whether N's will breach to form good contracts with both N's (double breaches) and M's or exclusively with M's (single breaches). Ceteris paribus, it is more advantageous, as we explain below, to form a contract with an M than with an N. Similarly, search is at least as profitable for an M as for an N. Thus there are three possibilities for search: it may be unprofitable for everyone, profitable for M's but not for N's, or profitable for both M's and N's. The last case subdivides in two breaching possibilities—either both single and double breaches are advantageous or only single breaches. The four possible behavior models are shown in Table 1. The effects of various meetings on the number of M's and N's are shown in Table 2. To interpret the first column of Table 2, we note that each of the h_M M's has a probability ah_M of meeting an M of the opposite type. p of these meetings result in poor matches, giving a flow of aph_M^2 new poor matches from meetings between M's. Each such meeting decreases the number of M's (of each type) by one and increases the number of N's by one. The equations determining the

TABLE 1

Behavioral Configurations

	Search by M's	Search by N's	Single breaches	Double breaches
A	yes	yes	yes	yes
B	yes	yes	yes	no
C	yes	no	irrelevant	
D	no	no	irrelevant	

[5] We have implicitly modeled contracting as instantaneous. Without instantaneous contracting, the assumption of no simultaneous meetings is an approximation.

TABLE 2

Numbers of Searchers, Quadratic Technology

Action	Poor match of M's	Good match of M's	Single breach	Double breach
Rate of flow	aph_M^2	$a(1-p)h_M^2$	$2a(1-p)h_M h_N$	$a(1-p)h_N^2$
Change in numbers of each type:				
without partners (h_M)	-1	-1	0	$+1$
in poor matches (h_N)	$+1$	0	-1	-2
in good matches	0	$+1$	$+1$	$+1$

number of searchers (see below) are calculated by multiplying the induced changes by the frequency of different types of meetings.

Under Configuration A, both M's and N's search, and any good match results in a contract's being signed. From Table 2 we can infer that the equations[6] are:

$$\begin{aligned} \dot{h}_M &= -ah_M^2 + a(1-p)\,h_N^2, \\ \dot{h}_N &= aph_M^2 - 2a(1-p)\,h_M h_N - 2a(1-p)\,h_N^2. \end{aligned} \tag{1}$$

The number of M's is diminished by matches between M's(ah_M^2) and enlarged by the individuals left partnerless when two N's make a good match $(a(1-p)\,h_N^2)$. The number of N's is increased by poor matches between M's (aph$_M^2$) and diminished by good matches between N's and M's $(2a(1-p)\,h_M h_N)$ and between N's $(2a(1-p)\,h_N^2)$. Notice that

$$\begin{aligned} \dot{h}_M \gtrless 0 \quad &\text{as} \quad h_M \lessgtr h_N(1-p)^{1/2}, \\ \dot{h}_N \gtrless 0 \quad &\text{as} \quad h_M \gtrless h_N((1-p)^{1/2} + (1+p)^{1/2})(1-p)^{1/2}\,p^{-1} \end{aligned} \tag{2}$$

Under Configuration B, behavior is the same as under A except that double breach does not occur. The equations are, therefore, the same as (1) after deleting all terms involving double breaches:

$$\begin{aligned} \dot{h}_M &= -ah_M^2, \\ \dot{h}_N &= aph_M^2 - 2a(1-p)\,h_M h_N, \end{aligned} \tag{3}$$

$$\begin{aligned} &\dot{h}_M < 0, \\ &\dot{h}_N \gtrless 0 \quad \text{as} \quad h_M \gtrless 2h_N\left(\frac{1-p}{p}\right). \end{aligned} \tag{4}$$

[6] All differential equations are equations in the mean, to avoid stochastic components. Thus we are assuming that numbers are sufficiently large to realize the expected number of meetings.

Under Configuration C, only M's search. Therefore the equations are:

$$\begin{aligned} \dot{h}_M &= -ah_M^2 < 0, \\ \dot{h}_N &= aph_M^2 > 0. \end{aligned} \tag{5}$$

Finally, under Configuration D, there is no search at all.

We define the positional values of being an M or an N, when the numbers of M's and N's are (h_M, h_N), as $V_M(h_M, h_N)$ or $V_N(h_M, h_N)$, respectively. An individual's positional value is the payoff that he can expect, given a correct forecast of the path of the economy and rational search and breach behavior. The equations of motion depend on the prevailing configuration and the numbers of M's and N's. The private decisions about search and breach that determine the configuration depend, in turn, on the future evolution of these equations of motion and the values of positions. Thus positional values are, indeed, functions solely of (h_M, h_N).

For positional value to be well-defined, we must describe how contracting works. Suppose that individuals i and j contemplate signing a contract which would yield them a combined positional value of $2V$. Let V^i and V^j be their current (i.e., pre-contract) positional values. Suppose further that i and j currently are in contracts which specify that they pay damages D^i and D^j, respectively, to their partners if they breach.[7] Then, we can define the surplus of the contemplated contract as $S = 2V - V^i - V^j - D^i - D^j$. We postulate that if i and j sign the contract, then they divide output and/or make side payments so as to share the surplus equally.[8] The individuals gain by signing the contract if and only if S is positive. Under our division rule, individuals i and j attain positional values $V^i + 1/2\, S$ and $V^j + 1/2\, S$, respectively, from the contract.

If, say, i breaches a contract with j, we assume that he pays j damages equal to $V^j - V_M$. (Recall that V_M is the positional value of being partnerless.) That is, j maintains the same expected payoff he had before the contract was breached. The damages are, therefore, compensatory. We focus on compensatory damage rules in this paper because (1) they constitute the basic principle for assigning damages under common law and (2) they are efficient in the limited sense that they ensure that a breach occurs if and only if there is an increase in the combined positional value of the principal affected parties[9] (the new partners and their original partners, if any). In Section 6 we consider equilibrium without damage payments. In our previous article we also considered liquidated damages, the damages that contracting parties themselves would choose.

[7] If i or j is currently partnerless, then $D^i = 0$ or $D^j = 0$, respectively.

[8] This division rule is also the Nash Bargaining Solution to the problem.

[9] For analysis of this effect of compensatory damages, see Mortensen [2].

In our simple two-quality model, two M's always wish to form a contract if they meet, since $V_N \geqslant V_M$ and $X > V_M$. Furthermore, with compensatory damages an N will breach his current contract only when he finds a good match, because only then can the surplus of his new contract be positive.

Notice that the incremental benefit that an M receives from any potential contract is larger than it would be were he an N, since he does not pay damages (which would diminish the surplus) and has a lower positional value. Therefore, the benefits of search are greater for M's than for N's. We need not consider, therefore, a behavior configuration in which N's but not M's search. Similarly, since an N would prefer to form a new contract with an M rather than another N, we can exclude configurations where double but not single breaches occur. We consider only equilibria where, at any moment, all individuals in the same position behave identically. Thus Configurations A, B, C, and D are, indeed, collectively exhaustive.

3. Positional Values, Trajectories and Boundaries in a Decentralized Market: Quadratic Technology

We assume that an individual maximizes positional value when deciding whether to search and breach. Positional value, however, depends on the future evolution of the economy and so must be determined by working backwards. We are interested in Nash equilibrium time paths. An equilibrium path specifies a behavior configuration at each instant of time and has the property that each individual finds the behavior prescribed for him optimal given the specified behavior of the others. To calculate the evolution of an equilibrium path, we derive and solve differential equations for positional values in each of the configurations. We then examine which configuration at any instant is consistent with equilibrium. When several configurations are all consistent at some instant—i.e., when there are multiple equilibrium paths—we select the configuration involving the most search and breach. So, for example, we select Configuration A over B, C, or D. We assume everyone is partnerless at the start,[10] i.e., h_N is zero, and h_M, arbitrary.

We begin the analysis with Configuration C. We refer to the set of (h_M, h_N) pairs where Configuration C behavior occurs on some equilibrium path as Region C.[11] Only M's search in Region C, and, therefore, along a Region C trajectory, h_M is steadily declining, while h_N is increasing at only p times the rate of h_M's decrease. Because M's make better partners than N's the gains from search monotonically decline for both M's and N's. Thus, a

[10] In the Appendix we drop this assumption.

[11] Regions A, B, D are similarly defined. Each point in $h_M - h_N$ space is, by our assumption of unique paths, associated with a unique region.

transition from Region C to either A or B is impossible. Once all N's stop searching, they will never wish to resume. Consequently, V_M^C, the positional value of an M in Region C, depends only on h_M. In Region C an M incurs search costs $c\Delta t$ in a small interval of time Δt and finds a partner with probability $ah_M\Delta t$. The value of a position at t equals the expected positional value at $t+\Delta t$, less search costs. Thus

$$\begin{aligned} V_M^C(h_M(t)) = &- c\Delta t + ah_M \Delta t(pX' + (1-p)X) \\ &+ (1 - ah_M \Delta t)\, V_M^C(h_M(t+\Delta t)), \end{aligned} \tag{6}$$

where $pX' + (1-p)X$ is the expected output from a match. Rearranging terms, defining $\Pi = pX' + (1-p)X$, letting Δt tend to zero, and substituting for $\dot{h}_M$ using (5) yields

$$-(ah_M^2)\frac{dV_M(h_M)}{dh_M} = c - ah_M\Pi + ah_M V_M^C(h_M). \tag{7}$$

Equation (7) completes the first piece of the analysis: calculation of the change in positional value in Region C.

Because h_M declines steadily in Region C, it ultimately reaches h_M', where the gain from search for the next instant is zero. At this point the search cost equals expected gross gain:

$$c = ah_M'\Pi. \tag{8}$$

Thus, search ceases at h_M', and the line $h_M = h_M'$ serves as the transition boundary between Regions C and D.[12] To find V_M^C, therefore, we solve (7) with terminal condition $V_M^C(h_M') = 0$:

$$V_M^C = \Pi - \frac{c}{a}\frac{\ln h_M}{h_M} + \frac{G}{h_M}, \tag{9}$$

where $G = (c/a)(\ln c/a\Pi - 1)$.

Next consider possible transitions from B to C. In Configuration B N's make new deals only with M's. Thus, the transition boundary separating B from C falls at that critical number of M's h_M'', where an N finds search just barely profitable:

$$c = ah_M''(1-p)(X - X'), \tag{10}$$

[12] We refer to the locus of possible transitions as the transition boundary. Since (as we shall see) part of this locus may be in Region A or, alternatively, may not be reachable from an initial position on the h_M axis, the actual boundary of Region C is a subset of the transition boundary.

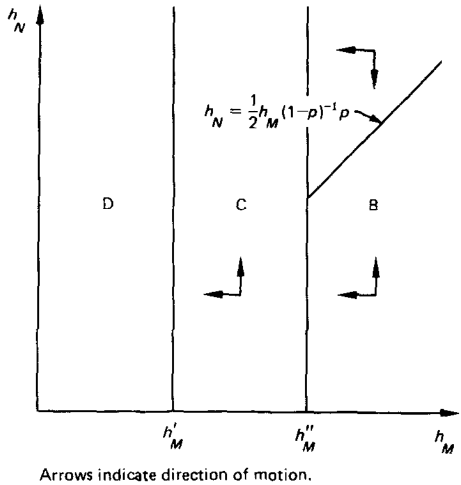


FIG. 1. Directions of motion in Regions B, C, and D.

where $X - X'$ is one half the surplus of a contract between an M and an N when further search by N's is unprofitable. To understand this equation, note first that the value of the N position is X' since N's do no further search once Region C is reached. Thus, damages are $X' - V_M$, and the surplus from a new contract is $(2X - V_M - V_N - D) = (2X - 2V_N) = 2(X - X')$. Notice that $(1-p)(X - X') < \Pi$. Therefore $h''_M > h'_M$, and transition borders appear as in Fig. 1, where D borders only on C, and C only on B.

Next, consider positional values in Region B. Suppose that an M searches for time Δt beginning at time t. With probability $ah_M(1-p)\Delta t$, he will meet another M and form a good match, giving value X. If he meets an M and forms a poor match (probability $aph_M \Delta t$) his positional value becomes V_N. If he encounters an N with whom he makes a good match (probability $a(1-p)\,h_N \Delta t$) his positional value is V_M plus one-half the surplus of $2X - V_M - V_N - D = 2X - 2V_N$. Otherwise his positional value is $V_M(h_M(t + \Delta t), h_N(t + \Delta t))$. Thus, we have

$$\begin{aligned} &V_M^B(h_M(t), h_N(t)) \\ &\quad = -c\,\Delta t + a\,\Delta t\, h_M(1-p)X + a\,\Delta t\, h_M\, pV_N^B(h_M(t+\Delta t), h_N(t+\Delta t)) \\ &\qquad + a\,\Delta t\, h_N(1-p)(X - V_N(h_M(t+\Delta t), h_N(t+\Delta t)) \\ &\qquad + V_M(h_M(t+\Delta t), h_N(t+\Delta t))) \\ &\qquad + (1 - a\,\Delta t\, h_M - a\,\Delta t\, h_N(1-p))\, V_M(h_M(t+\Delta t), h_N(t+\Delta t)). \qquad (11) \end{aligned}$$

For an N, only a meeting with an M where a good match is made changes positional value. Positional value becomes $V_N + \frac{1}{2}(2X - 2V_N) = X$. Thus we have

$$V_N^B(h_M(t), h_N(t)) = -c\,\Delta t + a\,\Delta t\, h_M(1-p)X + (1 - a\,\Delta t\, h_M(1-p)) \times V_N^B(h_M(t+\Delta t), h_N(t+\Delta t)). \tag{12}$$

Taking limits we obtain the pair of differential equations

$$\begin{aligned}\frac{dV_M^B}{dt} &= c - a(1-p)(h_M + h_N)X - a(ph_M - (1-p)\,h_N)\,V_N^B + ah_M\,V_M^B,\\ \frac{dV_N^B}{dt} &= c - a(1-p)\,h_M X + a(1-p)\,h_M V_N^B.\end{aligned} \tag{13}$$

Using these equations, we can conclude that a transition from Region B to A is impossible. The surplus from a double breach is $S = X + V_M - 2V_N$. In Region B, S is negative since double breaches are unprofitable. Furthermore,

$$\frac{dS^B}{dt} = \frac{dV_M^B}{dt} - 2\frac{dV_N^B}{dt} = -c - a(X - V_N^B)((1-p)\,h_N + ph_M) + ah_M S < 0. \tag{14}$$

Therefore, the double breach surplus never becomes positive, and movement from B to A is ruled out.

Because transition from B to A is impossible and since $h_M'' > h_M'$, any trajectory crossing Region B must then move into Region C. (See Fig. 1.) Thus, once Region B is reached, N's never again contract with other N's and so V_N becomes a function of h_M alone. We can calculate positional values in B by solving the differential equation pair (13) with terminal conditions $V_M^B(h_M'', h_N) = V_M^C(h_M'')$ and $V_N^B(h_M'') = X'$. For $p \neq \frac{1}{2}$ we obtain

$$\begin{aligned}V_M^B(h_M, h_N) &= X - \frac{c(1-p)\ln h_M}{a(1-2p)\,h_M} + \frac{H}{h_M}\\ &\quad - \frac{cp^2 h_M^{2p-2}}{a(1-p)(1-2p)^2\,(h_M'')^{2p-1}}\\ &\quad + h_N\left[\frac{-c(1-p)\,h_M^{-2}}{ap(1-2p)} + \frac{ch_M^{p-2}}{ap(1-p)(h_M'')^p}\right.\\ &\quad \left. + \frac{cph_M^{2p-3}}{a(1-p)(1-2p)(h_M'')^{2p-1}}\right];\end{aligned} \tag{15}$$

$$V_N^B(h_M) = X + \frac{c}{aph_M} + \frac{J}{h_M^{1-p}},$$

where H and J are chosen so that the terminal conditions hold.

We now turn to an examination of Region A. For an M, the arrival of new opportunities in Region A follows the same rules as their arrival in Region B. Thus the differential equation for the change in value is the same. For an N, positional value falls more rapidly because of the added opportunity of double breaches. These occur at the rate $a(1-p)\,h_N$ and add $X - 2V_N^A + V_M^A$ to value when they occur. Thus the differential equations for values in Region A satisfy

$$\frac{dV_M^A}{dt} = c - a(1-p)(h_M + h_N)\,X - a(ph_M - (1-p)\,h_N)\,V_N^A + ah_M V_M^A$$

$$\frac{dV_N^A}{dt} = c - a(1-p)\,h_M(X - V_N^A) - a(1-p)\,h_N(X - 2V_N^A + V_M^A)$$

$$\begin{aligned} \frac{dS^A}{dt} &= \frac{dV_M^A}{dt} - 2\,\frac{dV_N^A}{dt} \\ &= -c + a(1-p)(h_M + h_N)S - a((1-p)\,h_N + ph_M)(V_N^A - V_M^A) \end{aligned} \tag{16}$$

$$\frac{d^2V_N^A}{dt^2} = a(1-p)(h_M + h_N)c + a^2(1-p)\,ph_M(h_M + h_N)(V_N^A - V_M^A) > 0.$$

We shall see that trajectories can move from Region A directly to any of the other three regions. However, there are two distinct patterns. For one set of parameter values, the equilibrium path moves from an initial position in A to B then C then D. For the remaining values, movement from A is directly

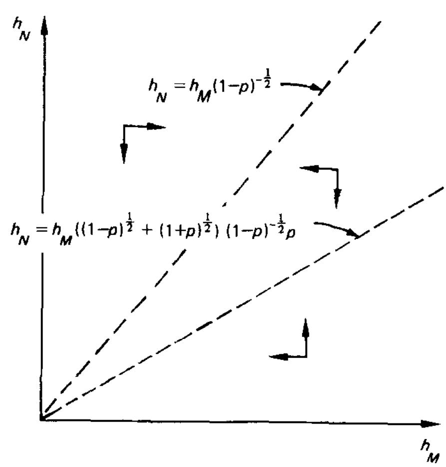


FIG. 2. Directions of motion in Region A.

to either C or D, and Region B does not exist. We first show that which of these two patterns applies depends on which region, A or B, contains the line $h_M = h''_M$. Then we consider, in turn, transitions from A to B, C, and D.

We observe from (2) that the direction of movement is as shown in Fig. 2. For an initial position on the h_M axis, the trajectory can never be to the left of the line $h_N = h_M(1-p)^{-1/2}$ while in Region A: movements across the line are not possible with Configuration A behavior. In the Appendix we briefly consider initial positions in this area. In the text we consider region boundaries only to the right of this line (although this restriction is often unstated).

Consider the point (h''_M, h_N) for some positive h_N. From Fig. 1 either this point is on the B–C border, or it is part of Region A. To check the former possibility, we need to evaluate the surplus from a double breach at (h''_M, h_N) using positional values in Region B. If the surplus is negative, (h''_M, h_N) is on the B–C border. If the surplus is positive, the point cannot lie in Region B. If (h''_M, h_N) is on the B–C border then, at this point, V_M equals $V^C_M(h''_M)$, V_N equals X', and we have

$$\begin{aligned} S &= 2X - 2V_N - 2D = 2X - 4V_N + 2V_M \\ &= 2\left(X - 2X' + \Pi - \frac{c}{a}\frac{\ln h''_M}{h''_M} + \frac{c}{ah''_M}\left(\ln\frac{c}{a\Pi} - 1\right)\right) \\ &= 2(X - X')\left(1 + (1-p)\ln\frac{(1-p)(X-X')}{X' + (1-p)(X-X')}\right) \leqslant 0 \end{aligned} \tag{17}$$

Since (17) does not depend on h_N, either all the points (h''_M, h_N) lie on the

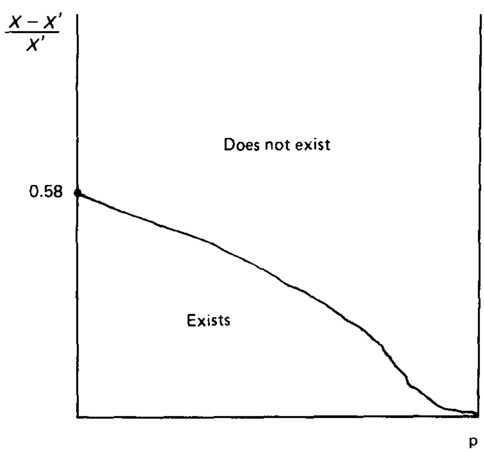


FIG. 3. Existence of Region B.

B–C border or they are all in Region A. Equation (17) is thus a necessary and sufficient condition for the existence of a B–C transition border.

Equation (17) indicates that for $X - X'$ sufficiently small relative to X' the surplus at $(h''_M h_N)$ is negative, and a B–C border exists. The smaller X relative to X', the less valuable is a good match and hence a double breach. Sufficiently small values of p also lead to a negative surplus. Decreasing p makes good matches easy to find, increasing the benefit to an N of remaining in the search market rather than taking advantage of a double breach opportunity. Therefore, small $X - X'$ (relative to X') or small p implies that Region B exists. These possibilities are shown in Fig. 3.

If (17) holds, we know that Region B exists and can integrate the value equations backward, ultimately reaching the A–B transition border. This border is defined as the locus where there is zero gain from a double breach:

$$X - 2V_N^B(h_M) + V_M^B(h_M, h_N) = 0. \tag{18}$$

Using Eqs. (15) and (16), we can write (18) as

$$\begin{aligned} h_N = h_M[&-(1-p)^2 + (1-2p)(h_M/h''_M)^p + p^2(h_M/h''_M)^{2p-1}]^{-1} \\ &\times [p(1-p)^2 \ln(h_M/h''_M) - 2(1-2p)(h_M/h''_M)^p \\ &+ p^3(1-2p)^{-1}((h_M/h''_M)^{2p-1} - 1) \\ &+ p(1-p)(1-2p)\ln((pX' + (1-p)X)/(1-p)(X-X')) \\ &+ (1-2p)(2-p)]. \end{aligned} \tag{19}$$

This locus is shown in Fig. 4. As h_M approaches h''_M, h_N increases without limit since $p^2 + (1-2p)$ equals $(1-p)^2$.

In Fig 4 we show the transition boundary. This suggests that Region A lies to the right, and B to the left, of locus (19). We have not confirmed that this is correct. To see the potential complication, consider the possibility that an equilibrium trajectory, when in Region A, might cross (19) more than once (see Fig. A1 and the discussion in the Appendix). If this were possible, only the last crossing would be a bonafide A to B transition, since we assumed the occurrence of the equilibrium path with the most breach. It would mean, furthermore, that Region A protrudes to the left of (19). We have not been able to rule out such multiple crossings. We can claim with accuracy, therefore, only that the A–B transition border is a subset of locus (19).

If S as defined by (17) is positive, the line $h_M = h''_M$ lies in Region A. A transition from B to A is impossible. Region B, if it exists, lies to the right of $h_M = h''_M$, and Regions C and D lie to the left. Therefore a positive S implies that no equilibrium path can go from Region B to another region, thus precluding the existence of B.

When B does not exist, we need to determine the A–C or A–D borders.

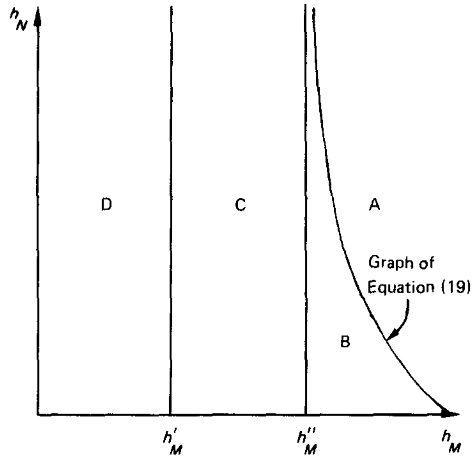


FIG. 4. Equilibrium regions when $X + V^C_M(h''_M) < 2X'$.

The A–C border is described by the curve showing indifference to continued search by N's:

$$c = a(1-p)\, h_M(X - X') + a(1-p)\, h_N(X - 2X' + V^C_M(h_M)). \tag{20}$$

The shape of the curve (20) depends on the sign of $X - 2X'$. Note that $c > a(1-p)\, h'_M(X - X')$. If $X < 2X'$, define h'''_M by $V^C_M(h'''_M) = 2X' - X$. Then $h'''_M > h'_M$, the A–C border lies to the right of the line $h_M = h'''_M$, and tends toward the line as h_N increases. This possibility is shown in Fig. 5. If $X > 2X'$, the A–C border reaches the line $h_M = h'_M$. In this case an A – D border exists and is given by

$$c = a(1-p)\, h_M(X - X') + a(1-p)\, h_N(X - 2X'). \tag{21}$$

This case is shown in Fig. 6. In Fig. 6 we have omitted indication of direction of movement in Region A. One may readily verify that the straight lines separating the areas of different direction of movement of Fig. 2 may bear any relation to the point of intersection of Regions A, C and D.

By analogy with the A–B border, we may inquire whether an equilibrium trajectory in Region A can cross the A–C ((20)) or A–D ((21)) transition borders more than once. Multiple crossings can be ruled out by the fact that an equilibrium trajectory is flatter than a 45 ° line ($dh_N/dh_M > -1$) (since the aggregate number of searchers is decreasing), whereas the transition are steeper than a 45° line: from implicit differentiation of (20) we have

$$\frac{dh_N}{dh_M} = -\frac{X - X' + h_N V'^C_M}{X - 2X' + V^C_M} < -1. \tag{22}$$

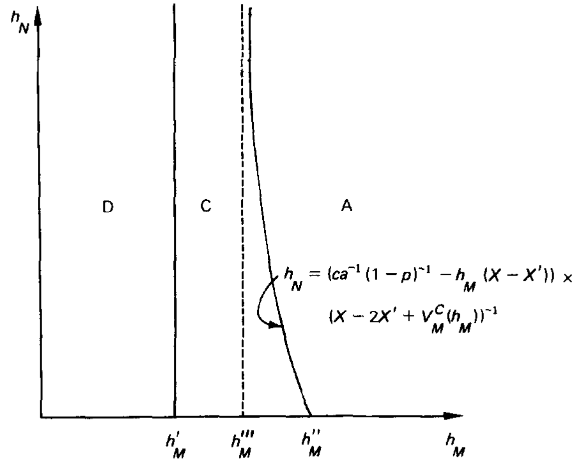


FIG. 5. Equilibrium regions when $X < 2X'$ and $X + V^C_M(h''_M) > 2X$.

The inequality follows since $V'^C_M > 0$ (see (9)) and $X' > V^C_M$ (since further search is not worthwhile). The same conclusion holds for the A–D border (21).

There is one remaining loose end to check. As one moves backward along trajectories in A, we must verify that search and double breach remain worthwhile. Moving backwards, both h_M and $(h_N + h_M)$ are increasing and so, therefore, is the return to search. Moving backward along a path, the surplus from a double breach cannot change sign (see (16)) since the other two terms are negative and dominate the term in S if S approaches zero.

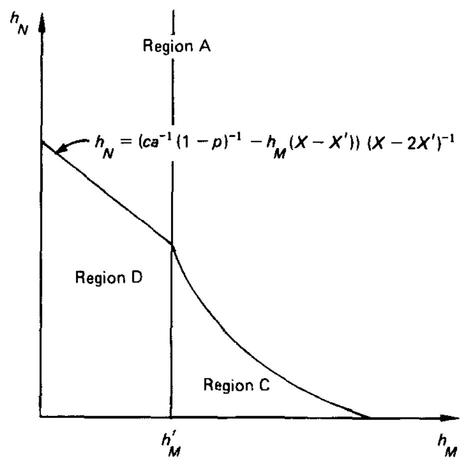


FIG. 6. Equilibrium regions when $X > 2X'$.

4. Inefficiency of Equilibrium

In Section 5, we describe efficient paths. In this section we examine the change in aggregate net output from perturbations of the equilibrium transition boundaries.[13] We show that the C–D boundary is efficient, but that shifts to the left of the B–C, A–C, and A–D borders (implying increased search) raise aggregate net output. The increase in breach resulting from a shift to the left in the A–B border also raises net output.

First consider the border between Regions C and D. If all M's search, the social gain per unit time is $ah_M^2(pX' + (1-p)X)$, while the social cost is ch_M.[14] Thus, the efficient C–D border, obtained by equating these expressions is the same as the competitive border. This coincidence may seem surprising, since an additional searcher creates an externality (an improvement in the positional value of other searchers) that does not seem to be captured by compensatory damages. The coincidence however is an artifice of the model's symmetry. The social gain from search is the sum of the individual gains. Since, under Configuration C, all searchers are identical, the social gain becomes zero precisely at the point where any individual gain vanishes. Thus the social and private incentives for search are the same.[15]

Next consider a slight shift to the left of the equilibrium B–C border. If the N's cease searching when $h_M = h_M''$, aggregate net output from those still in the market is $h_N X' + h_M'' V_M^C(h_M'')$. Suppose that N's continue to search an instant longer. The presence of N's following Configuration B behavior does not alter the time path of h_M, since single breaches do not affect the number of M's (Eqs. (3) and (5) are the same). Thus after the N's cease searching, the trajectory is the same as in Section 3. The cost of the N's additional search is ch_N per unit time. The additional output per unit time is the aggregate surplus from the resulting good matches, $2a(1-p)\, h_M h_N(X - X')$. The N's receive only half this surplus. Thus the private incentive to search is smaller than the social gain. A shift to the left of the B–C border (prolonging search by N's) raises net aggregate output since $c < 2a(1-p)\, h_M''(X - X')$.

We next consider perturbation of the equilibrium A–D border (which exists when $X > 2X'$). The border is the locus of points where N's are just willing to search (given that they receive half the surplus from both single and double breaches) and where M's are willing to search only if N's do so. M's find search worthwhile at the A–D border only because they receive part

[13] In doing these perturbations, we assume they do not affect earlier behavior before the perturbed boundary is reached.

[14] Aggregate output and search costs are twice these figures, but we continue to focus on one of the two types that make a pair.

[15] If individuals differed in search cost, all but the searchers with lowest cost would stop search too soon.

of the surplus from single breaches.[16] Again search ceases too soon—a shift to the left of the A–D border raises aggregate net output. The net gain from continued search is the surplus from matches between M's, from single breaches, and from double breaches, minus the search costs:

$$ah_M^2((1-p)X+pX')+2a(1-p)\,h_M h_N(X-X')$$
$$+\,a(1-p)\,h_N^2(X-2X')-c(h_M+h_N). \tag{23}$$

At the equilibrium border, (21), indifference of the N's to continued search implies a net social gain from continued search of

$$h_M(ah_M((1-p)X+pX')+a(1-p)\,h_N(X-X')-c) \tag{24}$$

But from the indifference of N's to search ((21), again) (24) becomes

$$ah_M(h_M+(1-p)\,h_N)\,X', \tag{25}$$

which is positive. To obtain the efficient A–D border, (23) is set equal to zero.

Consideration of the A–C border introduces a new element, not present in discussions of the other borders: the effect of double breach on the search environment as a result of changing the number of M's. The external effect is irrelevant at the A–D border because all search ceases there. Let V^* be the aggregate value of continued search. Once Region C is reached we have V^* equal to $h_M V_M(h_M)$. Since V_M^C is increasing in h_M, a double breach at the A–C border generates an external economy. Thus both single and double breaches have social values that differ from their private values to N's. The increase in aggregate value from continued search by N's of both types is the full value of single and double breaches plus the increased value of the search process for M's:

$$2a(1-p)\,h_M h_N(X-X')+a(1-p)\,h_N^2(X-2X'+V_M^C(h_M))$$
$$+\,a(1-p)\,h_N^2 h_M V_M'^C(h_M)-ch_N. \tag{26}$$

Using (20), (26) becomes

$$a(1-p)\,h_M h_N(X-X')=a(1-p)\,h_N^2 h_M V_M'^C, \tag{27}$$

which is positive.

Thus a leftward shift in the A–C border, resulting in prolonged search by

[16] M's find search at least as profitable as N's. Thus when N's are indifferent to search, as at the A–D border, M's strictly prefer to continue searching.

N's, is desirable in part because of the externalities from double breaches. For the efficient border we set (26) equal to zero.

We turn, finally, to the A–B border. A slight shift of the equilibrium border induces no change in search but affects breaching behavior. We show that the continuation of double breaches beyond the A–B border is worthwhile, assuming the rest of the equilibrium process is unchanged. At the A–B border, a double breach yields no private gain; nevertheless, there is a social gain. The value of net output of continued search in Region B is

$$V^* = h_M V_M^B(h_M, h_N) + h_N V_N^B(h_M). \tag{28}$$

A double breach creates a good match, adds one M and subtract two N's. The impact of these changes on aggregate value is

$$\begin{aligned} \Delta V^* &= X + \frac{\partial}{\partial h_M}(h_M V_M^B + h_N V_N^B) - 2\frac{\partial}{\partial h_N}(h_M V_M^B + h_N V_N^B) \\ &= \left(\frac{c}{a}\right)\left(\frac{p(2-p)}{1-p} + 2p\frac{h_N}{h_M}\right)\left(\frac{(h_M/h_M'')^{2p-1} - 1}{h_M(2p-1)}\right) \\ &\quad + \left(\frac{c}{a}\right)\left(\frac{1}{1-p} + \frac{h_N}{h_M}\right)\left(\frac{1}{h_M}\right) > 0, \end{aligned} \tag{29}$$

since $h_M > h_M''$ on the A–B border. (We have used (15) to calculate this derivative.)

To understand the externalities created by breach, we can examine the impact of a double breach on individuals other than the four principal parties (the two breachers and their partners). An M gains $X - V_N$, whereas an N gains $X - 2V_N + V_M$ from a good match with an N. If the double breach does not occur, the principals remains N's. If breach occurs, an M gains $X - V_M$ and an N, $X - V_N$, from a good match with the principal party left partnerless, while an M gains $V_N - V_M$ from a poor match with this party. Neither M nor N gains anything from meeting the breacher (who is now well matched). Thus the sign of an M's net gain from double breach is the same as that of

$$\begin{aligned} &p(V_N - V_M) + (1-p)(X - V_M - 2(X - V_N)) \\ &\quad = p(V_N - V_M) + (1-p)(2V_N - X - V_M); \end{aligned} \tag{30}$$

whereas the sign of the N's gain is the same as that of

$$(1-p)(X - V_N - 2(X - 2V_N + V_M)). \tag{31}$$

At the A–B border, $X - 2V_N + V_M$ is zero. Hence both (30) and (31) are positive there. Thus, when the principal parties are themselves indifferent about carrying out a double breach, the overall externality induced by such a breach is positive.

5. The Efficient Path

Since the C–D border is efficient, we can straightforwardly derive the efficient A–C and A–D borders from the perturbation analysis above. To complete the analysis, we verify that an efficient path can never cross from Region C to either A or B and that it never entails Configuration B behavior.

To describe the efficient A–C and A–D borders, we equate values of the perturbations of these borders with zero. Thus setting (23) and (26) equal to zero yields the border equations. In Figs 7, 8, and 9 we compare the efficient and equilibrium borders.

When $X > 2X'$ an A–D border exists and is obtained by setting (23) equal to zero:

$$a(1-p)\,X(h_M + h_N)^2 - 2a(1-p)\,h_N(h_M + h_N)\,X' + aph_M^2X' = c(h_M + h_N) \tag{32}$$

or

$$\begin{aligned} c = {} & a(1-p)\,h_M(X - X') + a(1-p)\,h_N(X - 2X') \\ & + aX'\left(p\frac{h_M^2}{h_M + h_N} + (1-p)\,h_M\right). \end{aligned} \tag{33}$$

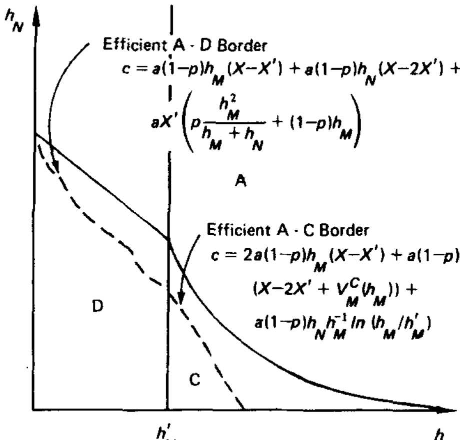


FIG. 7. Efficient and equilibrium borders when $X > 2X'$.

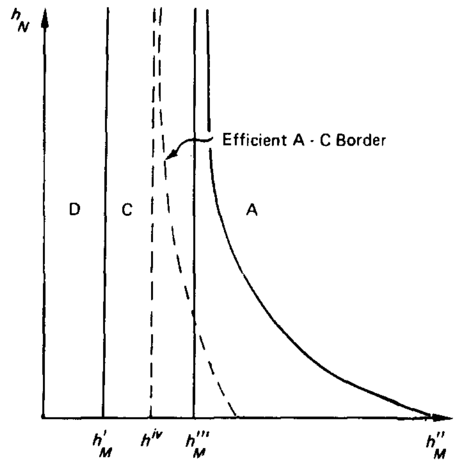


FIG. 8. Efficient and equilibrium borders when $X < 2X'$ and $X + V^C_M(h''_M) > 2X'$.

The equilibrium equation, (21), differs from (33); it does not contain the last term on the right. Thus for any value of h_N, there is a smaller value of h_M for the efficiency border than for the equilibrium border. For the A–C border, setting (26) equal to zero gives

$$c = 2a(1-p)\,h_M(X - X') + a(1-p)\,h_N(X - 2X' + V^C_M(h_M)) + c(1-p)\,h_N h_M^{-1} \ln(h_M/h'_M). \tag{34}$$

When $X > 2X'$, the coefficients for h_M and h_N are both positive and, the right hand side of (34) exceeds that of (20), the equilibrium border equation. Thus the efficient border lies to the left of the equilibrium border. This relation and the A–D border are shown in Fig. 7.

In Fig. 8 we illustrate the case where $X - 2X' + V^C_M(h''_M)$ is positive and $X < 2X'$. Substituting for V^C_M (from (9)) one sees that the coefficient of h_N in (34) vanishes at h^{iv}_M satisfying

$$\frac{h^{iv}_M}{h'_M} = \frac{pX' + (1-p)X}{2X' - X}. \tag{35}$$

Thus the efficient border is asymptotic to this line.

The remaining case to consider is where the market equilibrium has a Region B. In this case the efficient A–C border, like its equilibrium counterpart, is asymptotic to the line $h_M = h^{iv}_M$. This is shown in Fig. 9.

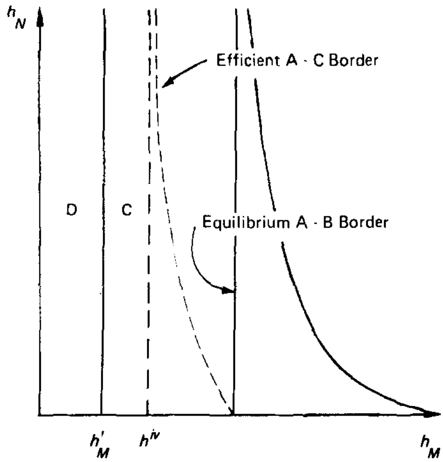


FIG. 9. Efficient and equilibrium borders when $X + V_M^*(h_M'') < 2X'$.

As the figures show, we have too little search in equilibrium unless $h_M(0)$ is in Region C or Region D.[17]

This discussion of efficient borders implicitly assumed that N's never resume search after having stopped, i.e., that the efficient path does not enter Regions A or B from Region C. We now prove that such transitions are impossible. In Region C, the dynamic programming value equation for aggregate net output is

$$\begin{aligned}\dot{V}^* &= \frac{\partial V^*}{\partial h_M}\dot{h}_M + \frac{\partial V^*}{\partial h_N}\dot{h}_N = -ah_M^2\frac{\partial V^*}{\partial h_M} + aph_M^2\frac{\partial V^*}{\partial h_N} \\ &= ch_M - a(1-p)\,h_M^2 X,\end{aligned} \tag{36}$$

where $V^*(h_M, h_N)$ is aggregate value from those not in good matches. In addition, we know that, in Region C, the marginal social value of N is constant, since N's simply accumulate:

$$\frac{d}{dt}\left(\frac{\partial V^*}{\partial h_N}\right) = 0. \tag{37}$$

From these two equations we can contradict the rise in the value of search by N's which would necessarily accompany a transition from C to either B

[17] We have assumed throughout that all N's stop searching at the same time. This assumption is justified since the social value of search by an N decreases when another N stops searching. That is, just to the right of the hypothesized borders, $\partial V^*/\partial h_N$ exceeds X', and, when these values are equal, it is socially worthwhile for all N's to stop searching simultaneously.

or A. Search by an N at time t has net value $2a(1-p)\,h_M(t)(X-\partial V^*/\partial h_N)-c$. In region C, $h_M(t)$ is decreasing and all other terms are constant. Furthermore $\partial V^*/\partial h_N < X$ because an N can, at best, make a good match. Thus the value of search declines, and so a C to B transition is impossible.

To consider a move from C to A, we must consider search by all the available N's. The aggregate return to search by N's is

$$2a(1-p)\,h_M h_N\left(X-\frac{\partial V^*}{\partial h_N}\right)+2a(1-p)\,h_N^2\left(X-2\frac{\partial V^*}{\partial h_N}+\frac{\partial V^*}{\partial h_M}\right)-ch_N.$$

Using (36) to eliminate $\partial V^*/\partial h_M$, we derive the return to search per N,

$$\begin{aligned}&2a(1-p)\,h_M\left(X-\frac{\partial V^*}{\partial h_N}\right)+2a(1-p)\,h_N\left(X-2\frac{\partial V^*}{\partial h_N}+(ah_M)^{-1}\right.\\&\quad\times\left.\left(aph_M\frac{\partial V^*}{\partial h_N}+a(1-p)\,h_M X-c\right)\right)-ch_N\\&=2a(1-p)\,h_M\left(X-\frac{\partial V^*}{\partial h_N}\right)+2a(1-p)\,h_N(2-p)\left(X-\frac{\partial V^*}{\partial h_N}\right)\\&\quad-c(2(1-p)\,h_N h_M^{-1}+h_N).\end{aligned}\tag{38}$$

Differentiating with respect to time (and using (37)) we have

$$\begin{aligned}&2a(1-p)\left(X-\frac{\partial V^*}{\partial h_N}\right)(\dot h_M+(2-p)\,\dot h_N)\\&\quad-c(2(1-p)((h_M\dot h_N-h_N\dot h_M)/h_M^2)+\dot h_N)\\&=2a^2(1-p)\left(X-\frac{\partial V^*}{\partial h_N}\right)(-h_M^2+(2-p)\,ph_M^2)\\&\quad-ac(2(1-p)(ph_M+h_N)+ph_M^2)<0,\end{aligned}\tag{39}$$

since $(2-p)\,p<1$. Thus the return to search by N's decreases per N while in Region C and, therefore, can never become positive.

We now turn to the proposition that the efficient trajectory never involves Configuration B behavior. We show that there is a higher net output flow from either Configuration A or C behavior if there is a border where the efficient trajectory leaves Region B[18]. This contradicts the possibility of Configuration B behavior on the efficient path. First consider the B–C

[18] Nor can it stay indefinitely in Region B.

border. The excess of aggregate net output flow that accrues from Configuration B behavior above that yielded by Configuration C is the additional output from single breaches less the search cost of N's,

$$2a(1-p)\,h_M h_N\left(X-\frac{\partial V^*}{\partial h_N}\right)-ch_N. \tag{40}$$

If a B–C efficient border exists, it is the locus where (40) equals zero. Since N's search no further, $\partial V^*/\partial h_N = X'$ at the border. Therefore the border is defined by the equation $h_M = \hat{h}_M$, where

$$\hat{h}_M=\frac{c}{2a(1-p)(X-X')}. \tag{41}$$

The difference between the Congifuration A and B rates of aggregate net output is the social gain from double breaches:

$$a(1-p)\,h_N^2\left(X-2\frac{\partial V^*}{\partial h_N}+\frac{\partial V^*}{\partial h_M}\right). \tag{42}$$

At the B–C efficient border, $\partial V^*/\partial h_N = X'$ and $\partial V^*/\partial h_M = V_M^C(\hat{h}_M) + \hat{h}_M(dV_M^C(\hat{h}_M)/dh_M)$ where V_M^C is given by (9). Thus, at the border,

$$\begin{aligned} X-2\frac{\partial V^*}{\partial h_N}+\frac{\partial V^*}{\partial h_M} &= X-2X'+\Pi-\frac{c}{a\hat{h}_M}\\ &= X-2X'+\Pi-2(1-p)(X-X')\\ &= p(X-X')>0. \end{aligned} \tag{43}$$

Formula (42) is therefore positive, and so an efficient B–C border does not exist. That is, when the social return to search by N's is close to zero and that of M's is positive, double breach is socially worthwhile.

An efficient B–D border is similarly ruled out by the positive social value of double breaches. At a B–D border, $\partial V^*/\partial h_M = 0$, $\partial V^*/\partial h_N = X'$, and the gain from double breach, (42), is

$$a(1-p)\,h_N^2(X-2X). \tag{44}$$

At this border the additional net output from search is just zero. That is,

$$ah_M^2\Pi+2a(1-p)\,h_M h_N(X-X')=c(h_M+h_N). \tag{45}$$

Furthermore, search by N's must be socially worthwile at the border. That is, Configuration B must be more efficient than C, implying $h_M \geqslant \hat{h}_M$ or

$$2a(1-p)\,h_M(X-X')\geqslant c. \tag{46}$$

Subtracting h_N times (46) from (45), we obtain

$$ah_M^2 \Pi \leqslant ch_M. \tag{47}$$

From the condition $h_M \geqslant \hat{h}_M$, (47) becomes

$$c \geqslant a\hat{h}_M \Pi = \frac{c\Pi}{2(1-p)(X-X')}$$

or

$$2(1-p)(X-X') \geqslant (1-p)X + X'. \tag{48}$$

This last expression implies $X > 2X'$, implying a positive gain from double breaches ((44)), which is a contradiction.

The last remaining possibility is a transition from B to A. At the A–B border, the surplus from double breaches must be zero; hence

$$X - 2\frac{\partial V^*}{\partial h_N} + \frac{\partial V^*}{\partial h_M} = 0. \tag{49}$$

Under Configuration B, the dynamic programming value equation is

$$\begin{aligned}\frac{d}{dt} V^*(h_M(t), h_N(t)) &= \frac{\partial V^*}{\partial h_M}\dot{h}_M + \frac{\partial V^*}{\partial h_N}\dot{h}_N \\ &= -ah_M\frac{\partial V^*}{\partial h_M} + (aph_M^2 - 2a(1-p)\,h_M h_N)\frac{\partial V^*}{\partial h_N} \\ &= c(h_M + h_N) - a(1-p)(h_M^2 + 2h_M h_N)\,X. \end{aligned} \tag{50}$$

Solving for $\partial V^*/\partial h_N$ using (49) and (50) we obtain

$$\begin{aligned}-(a(2-p)\,h_M^2 + 2a(1-p)\,h_M h_N)\frac{\partial V^*}{\partial h_N} = c(h_M + h_N) - a((2-p)\,h_M^2 \\ + 2h_M h_N(1-p)X,\end{aligned} \tag{51}$$

or

$$X - \frac{\partial V^*}{\partial h_N} = \frac{c(h_M + h_N)}{a((2-p)\,h_M^2 + 2(1-p)\,h_M h_N)} < \frac{c}{2a(1-p)\,h_M}. \tag{52}$$

If Configuration B is at least as efficient as C, however, then

$$X - \frac{\partial V^*}{\partial h_N} \geqslant \frac{c}{2a(1-p)} \tag{53}$$

(i.e., non-negative value of search by N's). Since inequalities (52) and (53) are mutually contradictory, we conclude that there is no efficient Region B.

6. Equilibrium without Damages

We have considered equilibrium assuming an idealization of the common law's provision of compensatory damages. Often individuals do not avail themselves of these damages. One way of modeling this behavior is to assume that individuals do not sign contracts unless they stop searching. Instead, we assume that after an individual has found a poor match, he can fallback on that match when further search is unprofitable, provided the fallback partner is still available. We assume that only one fallback contact is preserved, and that individuals do not replace an earlier fallback with a later one as long as the earlier one is available.[19] This behavior is captured by the model in Sections 2 and 3 if damages are always set at zero. We assume that after two individuals make a poor match, the decision to stop searching and to complete the project is jointly made. If the search decision were not joint, each partner would find search individually profitable, assuming his fallback partner did not search, at the point where search becomes jointly unprofitable.

The C–D border is the same with or without damages since only M's are involved. Thus $V_M(h_M)$ is the same in Region C as previously. Region B does not exist, because a double breach for a good match is always profitable with zero damages. Thus we are interested in A–C and A–D transitions. We shall see that the absence of damages lowers the incentive to search. Damage payments come out of surplus before its division. Thus a new partner effectively pays half the damages to one's old partner. This monopoly power over new partners serves as a further incentive to search for the original partners when damages are positive.[20]

In Configuration A search by a pair of N's for additional time Δt costs each $c\,\Delta t$. Each has probability $a(1-p)\,h_M\,\Delta t$ of forming a good match with an M, yielding a gain to the pair of one-half the surplus, $\frac{1}{2}(2X - V_M - V_N)$, less the damages suffered (but not paid) of $V_N - V_M$. Each partner has probability $a(1-p)\,h_N\,\Delta t$ of a double breach with surplus $\frac{1}{2}(2X - 2V_N)$, and the same damages, $V_N - V_M$. When the pair is just willing to search, V_N equals X', and we have the A–C border equation

$$c = a(1-p)\,h_M(X + \tfrac{1}{2}V_M - \tfrac{3}{2}X') + a(1-p)\,h_N(X - 2X' + V_M). \quad (54)$$

[19] These two assumptions are discussed below.

[20] This theme is explored in our previous paper, which also examines the damages which partners would choose to set (liquidated damages).

Because new partners do not share in damage payments, Eq. (54) differs from the A–C border with compensatory damages, (18), by $-a(1-p)\,h_M\frac{1}{2}(X'-V_M)$. (Symmetry implies no change in the gain from double breaches.) Thus there is less incentive to search without damages and the A–C border lies to the right of its position with damages. We can derive the A–D border from the A–C boder by setting V_M equal to zero. Again, there is less search than with compensatory damages.

The behavioral assumptions in this section have permitted a simple modification of the basic model. This modification illustrates the search incentive inherent in damages. With a further modification, we can illustrate the role of compensatory damages in joint maximization by partners. We have assumed so far that there were no single breaches for the sake of replacing one poor match with another. Yet individuals have an incentive to do so. When two M's form a poor match, they plan to evenly divide the output, $2X'$, should they carry out the match. If one of these N's meets a new M just as he is about to stop searching, the N can gain from breach even if his new partner is a poor match. Forming a new partnership, the N receives[21] $X'+\frac{1}{2}(X'-V_M)$ while the M receives $V_M+\frac{1}{2}(X'-V_M)$. The breach reduces the positional value of the previous partner from X' to V_M. Thus the aggregate positional value of the original partners has declined by one-half the compensatory damages that are not being paid. If the original partners have no way to control this inefficient breaching behavior, they will find search less profitable. The first order condition for the end of search will differ from that in the basic model, (18), by $-\frac{1}{2}ah_M(X'-V_M)$ rather than the factor $-\frac{1}{2}a(1-p)\,h_M(X'-V_M)$ in the first modification.

It is artificial to assume that individuals keep track of only one fallback partner. So too, in the basic model it is artificial to assume that individuals do not keep track of potential partners they have met with whom they do not form partnerships. Introducing a more complicated information structure would be interesting but would add considerably to the difficulty of analyzing the model.

7. Linear Technology

When the density of potential trading partners is low, the quadratic technology may be a reasonable approximation. However, when the density is high or the information about location is good, a searcher's problem is less one of finding a potential partner than of finding one who yields a high surplus. Such a situation can be approximated by assuming a constant

[21] Just as N's are due to stop searching, $V_N=X'$. The surplus from breach, V_N-V_M is positive everywhere, however, not just at the A–C border.

probability of meeting someone at all, independent of the numbers of potential partners (although constancy is improbable if the numbers of searchers are small). Analysis is quite different from that above since the market possibilities do not alter as time (and the numbers of searchers) changes.[22] Thus the entire $h_M - h_N$ space is characterized by a single configuration, A, C, or D, depending on parameters.[23] That is, individuals search until they find a good match, or search until their first match, or do not search at all, What is more, the equilibrium path is efficient.

There are three possibilities:

$c > a\Pi$	search is not worthwhile (Region D),
$a\Pi \geqslant c \geqslant a(1-p)(X-X')$	search is worthwhile for M's but not N's (Region C),
$c < a(1-p)(X-X')$	search is worthwhile for N's, implying that no bad matches are made; $V_N = V_M$ (Region A).

With this technology and behavior, positional values are independent of the numbers of searchers, giving

$$V_M^D = 0,$$

$$V_M^C = \Pi - \frac{c}{a},$$

$$V_M^A = V_N^A = X - \frac{c}{a(1-p)}.$$

Thus aggregate net output, V^*, is linear in h implying that the competitive process is efficient.[24]

8. Brief Summary

We have studied an allocation mechanism in which a searcher's meeting opportunities arrive according to a Poisson process. In Configuration A

[22] This result would change if those with poor contracts searched and the model were changed so that poor contracts were sometime carried out.

[23] If search continues until a good match is made, a poor match is of no additional value over being partnerless. Thus the damages are zero, and double breaches are always profitable, implying that Configuration B behavior does not occur.

[24] The efficiency of the competitive process under the linear technology is not robust to generalizations of the model. (See our earlier paper.)

under the quadratic technology, for example, poor opportunities arrive at the rate $aph_M(t)$ and good ones at $a(1-p)(h_M(t)+h_N(t))$. The values of these opportunities are determined endogenously; they depend on the evolution of the allocation process. The first part of the paper examines equilibrium evolutions: time paths where search and breach decisions are individually optimal.

An equilibrium time path consists of a sequence of behavior configurations determined by the parameters of the search technology (a, c), of tastes (p, X, X') and of initial position $(h_M(0))$. Section 3 enumerates all possible sequences. It demonstrates for example that the only equilibrium paths involving all four behavior Configurations are paths beginning in Region A and proceeding in turn to B, C, and D.

The heart of the paper is the demonstration that under the quadratic technology, search and breach give rise to externalities that generally cause inefficiency in the market process. Search by an individual creates a positive economy for other searchers. Because this externality is uncompensated, in equilibrium, N's stop searching too soon for efficiency. Double breach also creates external economies; it alters search environment by replacing two N's by an M on each side of the market. Since (at a point where double breach is individually just worthwhile), a searcher prefers the probability of meeting an M to twice that probability of meeting an N, such a replacement is a positive externality. Therefore, equilibrium paths entail too little breach; i.e., the transition from Region A to B occurs too soon for efficiency.

Given the simplicity of our model, it is natural to question whether the results are robust. We believe that the conclusion that double breach induces a positive externality when it is marginally worthwhile for individuals is quite general; the same result obtains in a variety of other models we have explored, including one with a continuum of qualities. Moreover, it is not a result peculiar to the quadratic technology. It applies, for example, to linear models that are sufficiently general so that poor contracts are sometimes carried out. (This illustrates that our conclusion that equilibrium under the linear technology is efficient does not generalize). Unfortunately, the reasons underlying the result are too complicated to go into here; they will be the subject of a forthcoming paper.

Whether adding an additional searcher creates a positive or negative externality depends on the value of the added searcher as a potential partner and on the search technology. For all technologies that we have examined—quadratic linear, and everything in between—adding a searcher s has two potential effects: (1) to raise to total probability of finding a potential partner and (2) to reduce the probability of finding a potential partner other than s. In the quadratic technology, only effect (1) is present, and so additional searchers always induce positive externalities, no matter how complicated the model otherwise is. In the linear technology, only effect (2) operates, and so the sign of the externality depends on s's value as a partner: positive if s is relatively valuable (an M, in the model of this paper) and negative if not (an N in the present model). (Externalities of search in the linear technology are irrelevant to efficiency in the linear model of this paper but relevant in more general linear models.) In models intermediate between quadratic and linear, both effects pertain, and so again a searcher's potential value will determine the sign of the externality.

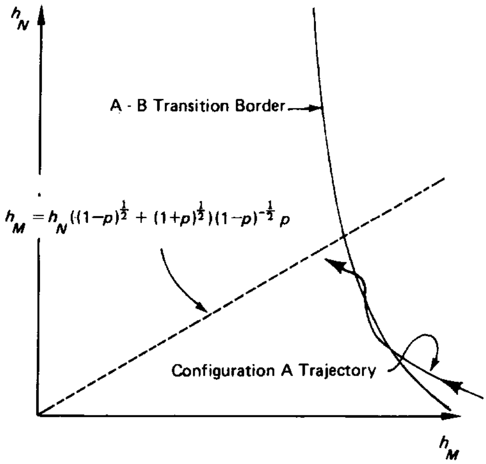
Appendix

In this appendix we discuss two issues: first, the possibility of multiple equilibria and, second, the nature of equilibrium paths from initial positions that are not on the h_M axis.

Whenever Nash equilibrium is the solution concept, as in this paper, the question of possible multiple equilibria arises. We avoided discussing multiplicity in the text by considering only the path with the maximum search and breach. If everyone else stops searching, the remaining individual must obviously find search unprofitable. Thus, taking any equilibrium path and altering it so that, at some arbitrary point, all individuals switch to Configuration D, we trivially generate a new equilibrium. (Of course, this change will require modification of earlier transitions.) Similarly, to the left of the line $h_M = h''_M$, an N's search is worthwhile only if other N's also search. Thus an equilibrium path following Configuration A between the line $h_M = h''_M$ and the A–C transition locus could, at any time, switch to Configuration C behavior and still remain an equilibrium trajectory. Indeed, an equilibrium path between these two curves could oscillate between Configurations A and C arbitrarily.

More interesting is the possibility of multiple equilibria involving A–B transitions. We have neither confirmed nor ruled out this possibility. If multiplicity were possible, a Configuration A trajectory would necessarily cross the A–B transition border as in Fig. A1. Anywhere on A–B transition border (more precisely, just to the left of the border), an N finds double breach unprofitable if everyone else follows Configuration B behavior. Therefore, any equilibrium path in Region A has an equilibrium continuation in Region B beginning at the A–B transition border. Suppose, however, that when an equilibrium path reaches the border, individuals persist with A behavior. The question is whether such behavior can be an equilibrium. The answer is yes if and only if the Configuration A trajectory from this point crosses the A–B transition border again.

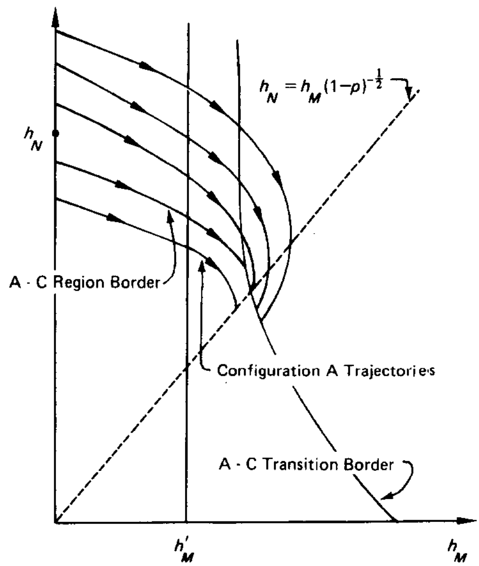
Now let us turn to equilibrium paths with initial positions not on the h_M axis. As long as the initial position lies below the line $h_M = h_N(1-p)^{1/2}$ (see (2)), the analysis is as before. Consider, therefore, the question of A–C tran-



AI

FIG. A1. Multiple equilibria at A–B transition border.

sitions when the initial position is above this line and when Region B does not exist. Fig. A2 shows the A–C transition border derived in the text (not yet shown, however, to be the region border in the present case) and a family of Configuration A trajectories. Moving backwards on one of these trajectories, the surplus from double breach remains positive as does an N's gains from search (see (16)). Therefore, Region A consists of all points to the left



A2

FIG. A2. A–C transitions for initial positions on the h_N-axis.

of the A–C transition border that lie above the trajectory just tangent to this border (see Fig. A2).

This analysis implies that positional values are not continuous in initial positions. As the initial position moves up the h_N axis, $V_N = X'$ until Region A is reached, at which point V_N increases discontinuously. The reason for this discontinuity is that equilibrium paths in Configuration A are impossible below Region A. Starting from a point just below this region, for example, an N's gain from search and double breach would be positive for awhile. However, if ever the configuration switches from A to C or D (as it must on an equilibrium path), the gains from search would be negative just before the transition, preventing such a path from being an equilibrium.

Acknowledgments

We gratefully acknowledge financial support from the NSF and mathematical assistance from Joel Yellin.

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