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On Imperfect Information and Optimal Pollution Control

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1. INTRODUCTION

The design of incentive schemes for implementing optimal plans in organizations where information differs across agents has been much studied in recent years; (see, e.g. Groves (1973), Weitzman (1974, 1978), Maskin (1977), Kwerel (1977) and the contributions to the *Review of Economic Studies* Symposium (1979)). One manner of classifying these studies is in terms of the number of rounds of communication between the planner and the rest of the organization. Such a classification is useful precisely because it is a pre-requisite for assessing the value of establishing additional information channels. In this note we shall provide such a classification in the context of a simple model of pollution control. By so doing we hope to place at least some of the contributions in a more general perspective.

We consider an economic environment consisting of n firms ($i, j = 1, \ldots, n$). Firm i faces a cost function $C(x_i, \theta_i)$, where x_i is the firm's pollution emission level ($x_i \in R^1_+$) and θ_i is a parameter (possibly a vector) known to the firm but not to the regulator. Let Θ_i be the set of possible values of θ_i, and take

$$\underline{\theta} = (\theta_1, \ldots, \theta_i, \ldots, \theta_n) \quad \text{and} \quad \underline{\theta}_{-i} = (\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_n).$$

Furthermore, let

$$\underline{x} = (x_1, \ldots, x_i, \ldots, x_n), \quad \text{and} \quad \underline{x}_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n).$$

In what follows we shall on occasion write $\underline{\theta} = (\theta_i, \underline{\theta}_{-i})$ and $\underline{x} = (x_i, \underline{x}_{-i})$.

Suppose that the social damage caused by the vector of pollution levels $\underline{x}$ is $D(\underline{x})$. We shall assume that the regulator can monitor $\underline{x}$ costlessly and that were the true value of $\underline{\theta}$ known to him he would wish to choose $\underline{x}$ to minimize the sum of damage and total costs $D(\underline{x})+\sum_{i=1}^{n} C(x_i, \theta_i)$. We assume that a unique minimum exists for every possible value of $\underline{\theta}$. Let $x_i^*(\underline{\theta})$ ($i = 1, \ldots, n$) be the optimal level of pollution for firm i. It is the full optimum; and it bears emphasis that x_i^* is a function of the vector $\underline{\theta}$. But by hypothesis, the regulator does not know, *a priori*, the true value of $\underline{\theta}$. One is therefore encouraged to search for optimal tax-subsidy schemes in the face of this initial lack of information. In particular, it is important to know whether the full optimum can be attained despite this lack. This is the question we shall examine in the next two sections. The last part of the paper considers what can be implemented in rather more general environments, where costs are not necessarily separable.

2. TWO-ROUND COMMUNICATION

Suppose that firms can communicate with the regulator but not with one another. We shall be concerned with schemes in which the regulator asks firms to reveal their cost functions (i.e. the θ_i's) to him and then imposes taxes on them which are based on the information supplied by them to him. Firms are informed in advance of the manner in which their messages will be translated into pollution taxes. The aim, therefore, is to devise schemes in which firms are induced to announce their true cost functions to the regulator.[1]

In a recent paper Kwerel (1977) exhibits such a scheme. In Kwerel's scheme the regulator issues a fixed number of transferable pollution licences and offers a subsidy for those licences which firms hold in excess of emissions. Both the number of licences to be issued and the subsidy rate offered are calculated on the basis of the cost information provided by firms. Kwerel shows that under such a scheme a firm will find it in its interest to tell the truth on the assumption that other firms are truthful; that is, truth-telling is rational for a firm provided it ascribes such rationality to others.

There are several limitations to this scheme. First, the procedure relies on perfect competition in the licensing market. It assumes implicitly, therefore, a large number of firms, and so may not be applicable to many industries. Second, the scheme becomes considerably more cumbersome, if not entirely unworkable, if one drops Kwerel's supposition that different firms' pollutants are perfect substitutes in the social damage function. Indeed, without perfect substitutability each firm would, in general, face a different price for licences. At the very least, these price differentials would set the scene for the emergence of a black market. Finally, although we do not feel that Kwerel's equilibrium notion (where truth-telling is rational if one believes others to be telling the truth) is too weak, a stronger form of equilibrium is available. It is possible to design procedures in which a firm's best strategy is to be truthful, regardless of the behaviour of others; that is, procedures in which truthfulness is a dominant strategy.

We seek tax functions

$$T_i: \prod_{i=1}^{n} \Theta_i \times R^1_+ \to R^1, \quad (i = 1, 2, \ldots, n)$$

such that, if $\hat{\theta}_i$ is firm i's actual cost parameter, then for any possible announcement θ_i it makes to the regulator, for any pollution level x_i it chooses, and for any possible announcements, $\underset{\sim}{\theta}_{-i}$ by other firms,

$$C(x_i, \hat{\theta}_i) + T_i(x_i, \theta_i, \underset{\sim}{\theta}_{-i}) \geqq C(x_i^*(\hat{\theta}_i, \underset{\sim}{\theta}_{-i}), \hat{\theta}_i) + T_i(x_i^*(\hat{\theta}_i, \underset{\sim}{\theta}_{-i}), \hat{\theta}_i, \underset{\sim}{\theta}_{-i}). \quad \ldots(1)$$

One tax scheme which satisfies (1), is a simple adaptation of the Groves (1973, Theorem 1), Clarke (1971) and Vickrey (1960) public choice mechanism. The tax functions are

$$T_i(x_i, \theta_i, \underset{\sim}{\theta}_{-i}) = D(x_i, \underset{\sim}{x}^*_{-i}(\theta_i, \underset{\sim}{\theta}_{-i})) + \sum_{j \neq i} C(x_j^*(\theta_i, \underset{\sim}{\theta}_{-i}), \theta_j) + A_i(\underset{\sim}{\theta}_{-i}), \quad \ldots(2)$$

where A_i is a firm-specific function.[2]

3. ONE-ROUND COMMUNICATION

We have just noted how, when firms are allowed to communicate with the regulator, the full optimum can be attained by employing the pollution tax schedules (2). We now suppose that the regulator cannot receive messages from firms. Thus, the regulator is required to impose tax-subsidy schedules without obtaining any information from firms. We must therefore describe the prior information that is possessed by the different agents in the organization.

Suppose that the θ_i's are perceived by everyone as independent random variables whose distributions are publicly known. Let E be the expectation operator. The regulator's objective is to minimize $E_\theta(D(\underline{x})+\sum_{i=1}^n C(x_i, \theta_i))$. Let $\tilde{x}_i(\theta_i)$ $(i = 1, ..., n)$ be a solution of the problem of minimizing expected costs (when θ_i is known, but $\underline{\theta}_{-i}$ is not), which are

$$C(x_i, \theta_i)+E[D(\tilde{x}_1(\theta_1), ..., \tilde{x}_{i-1}(\theta_{i-1}), x_i, \tilde{x}_{i+1}(\theta_{i+1}), ..., \tilde{x}_n(\theta_n))+\textstyle\sum_{j\neq i} C(\tilde{x}_j(\theta_j), \theta_j)].$$

In other words, $(\tilde{x}_1(\theta_1), ..., \tilde{x}_n(\theta_n))$, is a set of Bayesian team decision rules for firms which cannot send messages to the regulator. However, the organization we are studying in this note is not a team. Therefore, if $[\tilde{x}_1(\theta_1), ..., \tilde{x}_n(\theta_n)]$ is to be attained then we must identify tax functions T_i $(i = 1, ..., n)$, where T_i: $R^1_+ \to R^1$, such that for $x_i \in R^1_+$, $\theta_i \in \Theta_i$

$$C(x_i, \theta_i)+T_i(x_i) \geqq C(\tilde{x}_i(\theta_i), \theta_i)+T_i(\tilde{x}_i(\theta_i)) \qquad ...(3)$$

for $i = 1, ..., n$.

One tax scheme which satisfies (3) takes the form

$$T_i(x_i) = E[D(\tilde{x}_1(\theta_1), ..., \tilde{x}_{i-1}(\theta_{i-1}), x_i, \tilde{x}_{i+1}(\theta_{i+1}), ..., \tilde{x}_n(\theta_n)) + \textstyle\sum_{j\neq i} C(\tilde{x}_j(\theta_j), \theta_j)]+A_i(\underline{\theta}_{-i}),^3 \qquad ...(4)$$

where A_i is a firm-specific function.[4]

We emphasize that in contrast to the full optimum, $\tilde{x}_i$ is a function solely of θ_i. Social welfare, both ex-ante and ex-post, is therefore less under the decision rule $\underline{\tilde{x}}$ than under $\underline{x}^*$, except for one class of limiting cases.

Suppose, for example, that $D(\underline{x})$ is a symmetric function (e.g. $D(\underline{x}) = D\,(\sum_{i=1}^n x_i)$). Suppose also that the realized values of the θ_i's are random drawings from the same parent probability distribution. It should now be noted that $\underline{\tilde{x}} \approx \underline{x}^*$ if n is " large ", because the realized distribution of the θ_i's is approximately the same as the parent distribution. Since the latter is known publicly, the former is known *a priori* with a good deal of accuracy. That is, for " large " n, knowledge of the parent distribution is nearly sufficient, in the sense that the shortfall in expected social welfare due to limited communication possibilities, as a fraction of expected social benefit at the full optimum, is negligible. This limiting case has been studied by Radner (1972), Groves and Radner (1972), and Arrow and Radner (1979).

4. MORE GENERAL ENVIRONMENTS

So far we have been concerned with a social objective function of a very specific kind, namely, one where damages depend only on firms' pollution levels and where firms' costs enter separately and linearly. With such an objective function it is easy to see why the kind of tax schedules we proposed above lead to an optimum; in effect, they render the individual firm's objective identical to the social objective. When social objective functions are not of this kind, however, coincidence of individual and social ends may be impossible to attain. Nonetheless, in a large class of such cases, an optimum can still be reached.

We consider a social objective function of the general form $F(\underline{x}, \underline{\theta})$. We shall assume that both F and the private cost function C are twice continuously differentiable in all arguments. Let $x_i^*(\underline{\theta})$ be the socially optimum pollution level for firm i $(i = 1, 2, ..., n)$. That is, $(x_1^*(\underline{\theta}), ..., x_n^*(\underline{\theta}))$ solves the problem: $\max_{\underline{x}} F(\underline{x}, \underline{\theta})$. Assume that $x_i^*(\underline{\theta})$ is twice continuously differentiable in its arguments. For convenience, let us confine our attention to two rounds of communication. Then we must find tax functions

$$T_i\colon \textstyle\prod_{j=1}^n \Theta_j \times R^1_+ \to R^1, \quad (i = 1, ..., n)$$

such that for all $\hat{\theta}_i \in \Theta_i$, condition (1) holds. The n first order conditions are then

$$\frac{\partial C(x_i, \hat{\theta}_i)}{\partial x_i}\frac{\partial x_i^*}{\partial \theta_i}(\theta_i, \underline{\theta}_{-i})+\frac{\partial T_i}{\partial x_i}\frac{\partial x_i^*}{\partial \theta_i}+\frac{\partial T_i}{\partial \theta_i} = 0$$

at

$$\theta_i = \hat{\theta}_i \text{ and } x_i = x_i^*(\hat{\theta}_i, \underline{\theta}_{-i}) \quad \text{for } i = 1, ..., n. \qquad ...(5)$$

On using (5) we note that the (local) second-order conditions are

$$\frac{\partial x_i^*}{\partial \theta}(\theta_i, \underset{\sim}{\theta}_{-i}) \cdot \frac{\partial^2 C(x_i^*(\theta_i, \underset{\sim}{\theta}_{-i}), \theta_i)}{\partial x_i \, \partial \theta_i} \leqq 0, \quad i = 1, \ldots, n. \qquad \ldots(6)$$

(6) is, therefore, a necessary condition on the relationship between marginal cost and optimum pollution for the existence of a first-best tax scheme. It says, simply, that as the marginal cost of a firm rises, its optimum pollution level should fall. If, furthermore, (6) is a strict inequality, it is sufficient to guarantee the existences of taxes for which truth-telling is (locally) optimal for firms and leads to optimum pollution levels.

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NOTES

1. One could argue that if firms were not informed in advance of how the pollution taxes were to be assessed, they might reveal the truth in the absence of any other clearly defined strategy. But more likely there would be a strong pre-disposition towards firms exaggerating the cost of avoiding pollution.

2. Note that we have not supposed that the cost or damage functions are necessarily convex, an assumption required by Kwerel (1977). It should also be noted that the above procedure does not produce a balanced budget for the government. If a balanced budget is required we would need to give up the dominant-strategy requirement, and accept a Nash equilibrium. However, Kwerel's procedure has only the Nash property and does not balance the budget either.

3. Notice that the random variables θ_i must be independent for the expected damage here to be independent of θ_i, as required for these tax functions to work.

4. Weitzman (1978) has studied tax schemes such as those embodied in (4) under the assumption that $D(\underset{\sim}{x})$ and $C(x, \cdot)$ are all quadratic functions. In Weitzman (1974) the problem was further simplified by the insistence that $T_i(x_i)$ be either linear through the origin (i.e. a " price " control), or a step function equivalent to a quota restriction (a " quantity " control).

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