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RATIONAL EXPECTATIONS WITH IMPERFECT COMPETITION
A Bertrand–Edgeworth Example *

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We study a model in which price-setting oligopolists have private information about the commodity they sell. We construct an equilibrium in which some, but not all, of the information is transmitted by prices. We show that when the number of sellers increases, this equilibrium converges in distribution to the perfectly revealing competitive equilibrium.

1. Introduction

The literature on the transmission of private information by market prices has concentrated mainly on the competitive case. Two exceptions are Laffont and Maskin (1986) and Blume and Easley (1985). Specifically, Laffont and Maskin (1986) consider a model of a monopoly and a continuum of buyers in which the seller has private information about the quality of the good he sells. The monopolist can signal quality through his choice of price but, in general, gains by hiding information through the use of pooling strategies.

In the example of this paper we begin the extension of our analysis to oligopoly. We consider a model where oligopolists have fixed capacities and compete in prices. We exhibit a symmetric equilibrium in which transmission of information is imperfect, but show that, as the number of sellers grows, this equilibrium converges in distribution to the perfectly revealing competitive equilibrium.

Section 1 sets up the model and exhibits the unique symmetric mixed strategy equilibrium when there is perfect information. Section 2 discusses the case of duopoly under incomplete information, and section 3 presents the asymptotic results under incomplete information when the number of firms becomes large.

2. The model

Let θ be the quality of the commodity sold by the oligopolistic sector to a continuum [0, 1] of competitive consumers. Although firms can observe the value of θ, consumers cannot. Rather, they

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have prior probabilistic beliefs about θ represented by the uniform distribution on the interval [1, 2]. Demand is completely inelastic and equal to one at the price $p = \mathrm{E}\theta$, where 'E' denotes the expectation operator.

The oligopoly consists of n identical firms, each with capacity $(n+1)/n^2$. Production costs are supposed to be zero for simplicity, but the opportunity value of the commodity is $v(\theta) = \theta/3$ for each firm. Consequently firm j's objective function is

$$pq^j + \frac{\theta}{3}\left(\frac{n+1}{n^2} - q^j\right), \qquad j = 1, \ldots, n.$$

Notice that θ is common to all firms. That is, their private information is perfectly correlated.

Under perfect information, the competitive price, given our assumptions about demand and supply, would be $\theta/3$ and the monopoly price θ. Because of the capacity constraints, there exists no Bertrand equilibrium in pure strategies. However, there exists a symmetric mixed strategy equilibrium [see Dasgupta and Maskin (1986) and Allen and Hellwig (1986)]. Indeed, we can exhibit it explicitly. Let us assume that if the firms charging prices less than p ($< \theta$) sell a total quantity $x < 1$, then a firm setting p sells $\min\{1 - x, (n+1)/n^2\}$ (provided that it is the *only* firm charging p).

Proposition 1. Under perfect information, the unique symmetric mixed strategy equilibrium is characterized by the cumulative distribution function

$$F(p, \theta, n) = \left[\frac{3(n+1)p - (3+n)\theta}{3pn - \theta n}\right]^{1/(n-1)}.$$

Proof. For given θ, consider any price p in the support of the symmetric mixed strategy of firm j. The probability that p is the highest in the n-price sample is $F(p, \theta, n)^{n-1}$. In this case, firm j sells $1 - (n-1)[(n+1)/n^2] = 1/n^2$ and retains $(n+1)/n^2 - 1/n^2 = 1/n$. Its utility is $p/n^2 + \theta/3n$. If p is not the highest price [an event of probability $1 - F(p, \theta, n)^{n-1}$], firm j sells its whole endowment and obtains utility $[(n+1)/n^2]p$. Hence, firm j's expected utility is

$$\left[\frac{p}{n^2} + \frac{\theta}{3n}\right] F(p, \theta, n)^{n-1} + \frac{(n+1)p}{n^2}\left[1 - F(p, \theta, n)^{n-1}\right]. \tag{1}$$

The firm must derive the same expected utility from any price p in the support – in particular, from $p_{\mathrm{U}}(\theta, n)$, the upper bound. Moreover, there cannot exist an atom at $p_{\mathrm{U}}(\theta, n)$; otherwise, a firm would gain from charging a slightly lower price (thereby discontinuously increasing its clientele). Clearly, $p_{\mathrm{U}}(\theta, n)$ cannot exceed θ, since demand at such a price would be zero. But it cannot be less than θ because otherwise a firm could charge prices above $p_{\mathrm{U}}(\theta, n)$ without reducing sales.

We conclude that

$$p_{\mathrm{U}}(\theta, n) = \theta, \tag{2}$$

at which price a firm's expected utility is

$$\frac{\theta}{n^2} + \frac{\theta}{3n}. \tag{3}$$

Equating (1) and (3) gives us

$$F(p, \theta, n) = \left[\frac{3(n+1)p - (3+n)\theta}{3np - \theta n}\right]^{1/(n-1)}. \qquad \text{Q.E.D.}$$

3. Asymmetric information in the duopoly case

When $n = 2$, the formula of Proposition 1 reduces to $F(p, \theta, 2) = (9p - 5\theta)/(6p - 2\theta)$. We will show that when both firms play the strategy corresponding to this distribution, they remain in equilibrium even when θ is not observable by buyers.

Let $F^i(\cdot, \cdot, 2): R_+ \times [1, 2] \to [0, 1]$ be a strategy for firm i, $i = 1, 2$. It associates to each value of θ a cumulative distribution function $F^i(\cdot, \theta, 2)$ over prices. Although buyers do not know the value of θ, they know the firms' strategies. When they face prices p^1 and p^2, they compute their posterior distribution over θ to determine their optimal purchase. (If p^1 and p^2 are inconsistent with the firms' strategies, buyers will not be able to use Bayes' rule to obtain their posterior beliefs. But we suppose that they still have *some* beliefs.) Because θ is the same for both firms, buyers attempt to buy from the firm with the lower price. [However, they cannot buy more than $(n+1)/n^2$ from this firm.] Let $q^i(p^1, p^2)$ be the aggregate demand function facing firm i, $i = 1, 2$. In equilibrium, demand maximizes buyers' expected utility given their posterior beliefs and the capacity constraints [e.g., $q^1(p^1, p^2)$ may be positive even though $p^1 > p^2$ because consumers are rationed by firm 2].

A *perfect Bayesian equilibrium* (PBE) is a pair of strategies F^{*1}, F^{*2} and demand functions q^{*1} and q^{*2} such that for any θ and for any p^{*1} in the support of $F^{*1}(\cdot, \theta, 2)$, p^{*1} maximizes

(i) $$\int_{R_+} \left(p^1 - \frac{\theta}{3}\right) q^{*1}(p^1, p^2)\, \mathrm{d}F^{*2}(p^2, \theta, 2),$$

and similarly for firm 2,

(ii) $q^{*1}(p^1, p^2)$ and $q^{*2}(p^1, p^2)$ maximize buyers' expected utility given their posterior beliefs.

A simplifying feature of our example is that, for given beliefs, total demand is constant (and equal to 1) for all prices that are less than expected quality. Let $\mathrm{E}(\theta \mid p^1, p^2)$ denote buyers' expectation of quality conditional on p^1 and p^2 when each firm uses the strategy $F(p, \theta, 2) = (9p - 5\theta)/(6p - 2\theta)$. The following proposition shows that if p^1 and p^2 belong to the support of $F(\cdot, \theta, 2)$, the conditional expectation of θ exceeds both prices.

Proposition 2. For any p^1, p^2 *in the support of* $F(\cdot, \theta, 2)$, $\frac{9}{5}\min\{p^1, p^2\} \geq \mathrm{E}(\theta \mid p^1, p^2) \geq \max\{p^1, p^2\}$.

Proof. Given p^1, p^2, we know from the definition of F that $\frac{9}{5}\min\{p^1, p^2\} \geq \theta > \max\{p^1, p^2\}$. Hence, the result. Q.E.D.

To show that there exists a PBE where firms' strategies are F, we must specify what buyers believe when they encounter prices that are inconsistent with F. Let us suppose that, given such prices p^1 and p^2, they believe that the higher price exceeds expected quality but that the lower price (if less than 2) is less than expected quality.

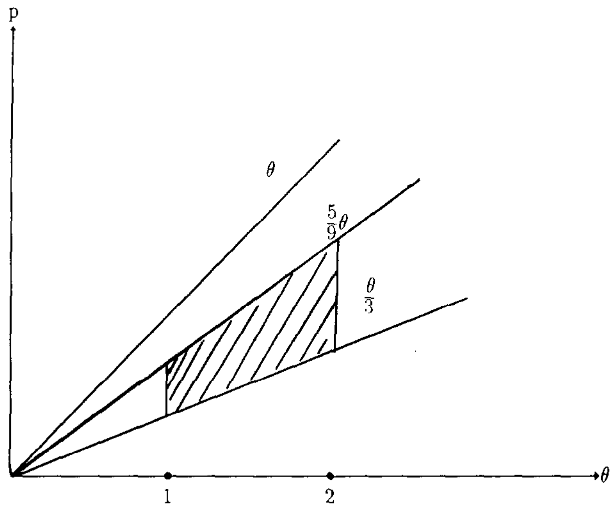

Fig. 1. A perfect Bayesian equilibrium (the shaded area corresponds to the support of the firms' strategy).

In view of Proposition 2, it remains only to demonstrate that, given these out-of-equilibrium beliefs, a firm does not profit by choosing prices below the lower endpoint of the support $p_L(\theta) = \frac{5}{9}\theta$ or above $p_U(\theta) = \theta$, given that the other firm behaves according to F. The former is clear because, at $p_L(\theta)$, a firm can already sell its entire capacity, so there is gain to lowering its price. To see the latter, suppose that, say, firm 1 charges a price $p^1 \geq \theta$. Given our assumptions about out-of-equilibrium beliefs, p^1 will exceed the buyers' expectation of quality with probability $F(\frac{5}{9}p^1, \theta, 2)$, Hence, because each firm's capacity is $\frac{3}{4}$, firm 1's expected improvement in its payoff (over the utility of selling nothing, which is $\theta/4$) is

$$\frac{1}{4}\left(p^1 - \frac{\theta}{3}\right)\left(1 - F\left(\frac{5}{9}p^1, \theta, 2\right)\right).$$

Substituting using the formula for $F(p, \theta, 2)$, we obtain

$$\frac{1}{4}\left(p^1 - \frac{\theta}{3}\right)\left(\frac{9\theta - 5p^1}{10p^1 - 6\theta}\right). \tag{4}$$

Now, the derivative of (4) with respect to p^1 is proportional to $30\theta p^1 - 25(p^1)^2 - 17\theta^2$, which is negative for all $p^1 \geq \theta$. We conclude that firm 1 does not gain by choosing a price above θ, and so $F^{*1} = F$ is part of a PBE after all. This equilibrium is illustrated in fig. 1.

Thanks to the indeterminacy of out-of-equilibrium beliefs, there are many other PBE's as well. For example, suppose that both firms set $p = \frac{3}{2}$ regardless of θ and that buyers believe that $E\theta = 1$ when faced with any other price. Of course, $E(\theta \mid \frac{3}{2}) = \frac{3}{2}$. This is clearly an equilibrium: if a firm raises its price, its demand drops to zero. Only by lowering its price to 1 can a firm sell more than $\frac{1}{2}$, but this would obviously be an unprofitable deviation.

Unfortunately, even in our simple example it seems an intractable problem to characterize all PBE's because, in general, an equilibrium is the fixed point of a fairly complicated mapping: firms' strategies give rise to consumers' beliefs, which determine their demand functions. But these demand functions, in turn, induce the firms' strategies.

4. Asymptotic results

We have shown that the perfect information symmetric equilibrium strategy $F(p, \theta, n)$ defines the firms' behavior in an equilibrium with incomplete information when $n=2$. We next show that the same is true for large values of n. To do so, we must again specify buyers' out-of-equilibrium beliefs.

Suppose that firms set prices $p^1, \ldots, p^n$. If these prices are inconsistent with the strategy $F(\cdot, \theta, n)$ for any θ, buyers believe that the highest price exceeds θ. If the remaining $n-1$ prices are still inconsistent with any θ, buyers believe that the second highest price exceeds θ, etc. Hence to show that firms will abide by the strategy $F(\cdot, \theta, n)$, it suffices, as in the case $n=2$, to establish that no firm will set a price above θ.

Proposition 3. If other firms play according to strategy $F(\cdot, \theta, n)$ and if buyers' out-of-equilibrium beliefs are as above, a firm obtains a higher expected payoff from setting $p=\theta$ than from $p>\theta$ provided that n is big enough.

Proof. Let $N(p, \theta, n)$ be the difference in a firm's payoff between setting $p \geq \theta$ and $p=\theta$ when the other $n-1$ firms behave according to $F(\cdot, \theta, n)$. Then,

$$N(p, \theta, n) \quad \text{is proportional to}$$

$$p\left[1-F(p_{\mathrm{L}}(p, n), \theta, n)\right]^{n-1}+\frac{\theta}{3}\left[1-\left(1-F(p_{\mathrm{L}}(p, n), \theta, n)\right)^{n-1}\right]-\theta$$

and

$$N(\theta, \theta, n)=0,$$

where $p_{\mathrm{L}}(p, n)$ is the lowest price consistent with p and $F(\cdot, \theta, n)$. Differentiating with respect to p, we obtain

$$\frac{\partial N}{\partial p}(p, \theta, n) \quad \text{is proportional to}$$

$$\left[1-F\left(\frac{p}{3} \frac{(3+n)}{(1+n)}, \theta, n\right)\right]^{n-1}$$

$$-(n-1)\left(p-\frac{\theta}{3}\right)\left[1-F\left(\frac{p}{3} \frac{(3+n)}{(1+n)}, \theta, n\right)\right]^{n-2} \frac{(3+n)}{3(1+n)} F'\left(p \frac{(3+n)}{3(1+n)}, \theta, n\right).$$

Hence,

$$\frac{\partial N}{\partial p}(p, \theta, n) \quad \text{is proportional to}$$

$$\left[1-F\left(\frac{p}{3} \frac{(3+n)}{(1+n)}, \theta, n\right)\right]-(n-1)\left(p-\frac{\theta}{3}\right)\left(\frac{3+n}{3(1+n)}\right) F'\left(\frac{p}{3} \frac{(3+n)}{(1+n)}, \theta, n\right).$$

As n becomes large, the first term on the right-hand side of the last formula tends to zero, whereas the second term tends to infinity. Hence, for n large enough, $\partial N/\partial p < 0$. Q.E.D.

As $n \to \infty$ the equilibrium prices of Proposition 3 converge in distribution to $\theta/3$. Hence as n becomes large, information about quality is transmitted increasingly accurately. Formally, we have

Proposition 4. *For any $\epsilon > 0$ and $\alpha > 0$, there exists n_0 such that for $n > n_0$, $F(\theta/3 + \alpha, \theta, n) > 1 - \epsilon$ uniformly in θ.*

Proof. It suffices to show that $F(\theta/3 + \alpha, \theta, n)^{n-1} > 1 - \epsilon$, uniformly in θ for n sufficiently large. The left-hand side of this last inequality is $[3(n+1)\alpha - 2\theta]/3n\alpha$ which converges to 1 uniformly as n grows. Q.E.D.

Note that as n grows, the equilibrium in which for all firms set $p^* = \frac{3}{2}$ persists. That is, the incomplete information enables firms to collude. It remains an open question whether reasonable constraints on out-of-equilibrium conjectures induce convergence of all PBE's to the competitive equilibrium.

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