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Uncertainty and entry deterrence*

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Summary. We study a model where capacity installation by an incumbent firm serves to deter others from entering the industry. We argue that uncertainty about demand or costs forces the incumbent to choose a higher capacity level than it would under certainty. This higher level diminishes the attractiveness of deterrence (Proposition 1) and, therefore, the range of parameter values for which deterrence occurs (Proposition 2).

Keywords and Phrases: Entry deterrence, Uncertainty, Capacity.

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1 Introduction

We study a model where capacity installation by an incumbent firm serves to deter others from entering the industry (c.f. Dixit, 1980; Spence, 1977; Schmalensee, 1981). We argue that uncertainty about demand or costs forces the incumbent to choose a higher capacity level than it would under certainty. This higher level diminishes the attractiveness of deterrence (Proposition 1) and, therefore, the range of parameter values for which deterrence occurs (Proposition 2).

Intuitively, uncertainty impedes the ability of the incumbent to commit itself to a high production level, if capacity must be installed before uncertainty is resolved. To deter entry, the incumbent must install enough capacity so that, if entry occurred, the entrant's profit would be zero (or negative). Under certainty, the incumbent will install no more capacity than it would use were entry to

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occur. [1] Suppose we now introduce uncertainty about demand so that mean inverse demand remains the same as under certainty (the logic is the same if instead or in addition there is uncertainty about costs). When demand is high, an incumbent that has installed the certainty level of capacity still continues to produce at capacity; price simply rises to reflect the higher demand. But when demand is low, the incumbent will wish to produce at less than full capacity. This means that the fall in price when demand is low is not so large as the rise in price when demand is high, and so if the entrant's profit is zero under certainty, it is positive with uncertainty. To deter entry, therefore, the incumbent must increase capacity above the certainty level to ensure that when demand is high it produces enough to drive the entrant's expected profit back down to zero.

The preceding argument is, of course, incomplete because it ignores the effects of uncertainty on (i) the entrant's behavior and (ii) the incumbent's behavior were it to accommodate entry. Nonetheless, it should provide a taste of the analysis to follow. We should mention that issues related to those of this paper have been studied by Gabszewicz and Poddar (1995).

2 The model

We suppose that to produce at all a firm must install at least k_o units of capacity. The cost of capacity is $q(>0)$ units per unit. A firm that installs k units of capacity ($k \geq k_o$) can produce up to k units of output. The marginal cost of output is $c(\tilde{\epsilon})$ where $\tilde{\epsilon}$ is a random variable. Inverse demand is given by the stochastic function $p(x, \tilde{\epsilon})$, where x is output. We suppose that p is twice differentiable, concave, and decreasing in x. Firms are risk neutral. These assumptions imply that a firm's profit function is concave in its own output.

There are two firms, an incumbent and a potential entrant. The incumbent moves first and selects its capacity level, k^1. The entrant then moves and either chooses to stay out of the market or else selects capacity k^2. After capacity is installed, firms observe the realization of $\tilde{\epsilon}$. They then choose output levels simultaneously. Of course, if the entrant has stayed out, its output is zero.

3 Results

Suppose first that $\tilde{\epsilon} \equiv 0$, i.e., that $c(\tilde{\epsilon}) = c^*$ and $p(\cdot, \tilde{\epsilon}) = p^*(\cdot)$. If k_d $(\geq k_o)$ is a capacity level for the incumbent that deters entry, then k_d satisfies

$$p^*(k_d + k_o) - q - c^* \leq 0 \,. \tag{1}$$

That is, provided that the incumbent produces at capacity (we examine below when this will be the case), setting $k^1 = k^d$ ensures that the entrant's profit is nonpositive when $k^2 = k_0$. [2]

[1] This is true, if output is determined by Cournot quantity-setting, but not, say, with Bertrand price-setting. Our propositions, however, should hold quite generally, as we argue in Section 3.

[2] If $k^2 > k_o$, the entrant's profit would be even lower, since $p^*(\cdot)$ is decreasing.

Now, for k_d to deter entry, it must be in the interest of the incumbent to produce up to capacity should entry occur; any excess capacity has no deterrent effect. Hence, k_d must satisfy

$$p^*(k_d + k_o) + k_d \frac{dp^*}{dk}(k_d + k_o) - c^* \geq 0\ ^3. \tag{2}$$

We assume that there exist values of $k_d (\geq k_o)$ satisfying conditions (1) and (2) simultaneously (otherwise, deterrence would be impossible). We claim that any such k_d successfully deters entry. To complete the argument we must show that, given that $k^1 = k_d$, it is not in the interest of the entrant to choose a capacity level greater than k_o in order to induce the incumbent to reduce its output below capacity. If the entrant chooses capacity $k^2 > k_o$, then (provided that the entrant's output equals k^2) [4] the incumbent chooses output no less than

$$\hat{x}^1(k^2) = \arg \max_{x^1 \leq k_d} \quad [p^*(k^2 + x^1)x^1 - c^* x^1] \, .$$

If k^2 is such that $\hat{x}^1(k^2) = k_d$, then obviously the incumbent is *not* induced to reduce its output. Assume, therefore, that $\hat{x}^1(k^2) < k_d$. Then

$$p^*(k^2 + \hat{x}^1(k^2)) + \hat{x}^1(k^2) \frac{dp^*}{dk}(k^2 + \hat{x}^1(k^2)) - c^* = 0 \, .$$

Differentiating this last equation implicitly with respect to k^2 and solving for $\frac{d\hat{x}^1}{dk}$, we obtain

$$\frac{d\hat{x}^1}{dk} = - \frac{\frac{dp^*}{dk} + \hat{x}^1 \frac{d^2p^*}{d(k)^2}}{2\frac{dp^*}{dk} + \hat{x}^1 \frac{d^2p^*}{d(k)^2}} \tag{3}$$

which is greater than -1.[5] Thus, increasing capacity induces a *less* than equal reduction by the incumbent. Because the entrant's profit is nonpositive when $k^2 = k_o$, its profit must be negative when $k^2 > k_o$ since the price of output would then be lower.

Thus, if k_d satisfies (1) and (2), it indeed deters entry. Let us suppose that any such k_d exceeds the *monopoly capacity*, i.e., the capacity level that the incumbent would install if there were no threat of entry (this level solves arg $\max[p^*(k)k - (c + q)k]$). Then, if k_d^* is the minimal capacity satisfying (1) and (2), k_d^* is the *optimal* entry-deterring capacity.[6] Notice that the left-hand sides of (1) and (2) are decreasing in k_d, and so (1) holds with equality at $k_d = k_d^*$.

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[3] Notice because p^* is decreasing and $k_o < k_d$, (2) implies that $p^*(k_d+k_o)+k_o \frac{dp^*}{dk}(k_d+k_o)-c^* >$ 0. Thus if the entrant installs capacity level k_o, it will produce at capacity.

[4] Without loss of generality, we can assume that this is the case: if it were equilibrium behavior for the entrant to produce $\hat{x}^2 (< \hat{k}^2)$ when $k^2 = \hat{k}^2$, then it would remain equilibrium behavior to produce $\hat{x}^2$ if capacity were reduced to $k^2 = \hat{x}^2$

[5] This is just the familiar result that Cournot reaction curves have slope greater than -1.

[6] We are leaving aside for now the question of whether it is actually in the incumbent's interest to deter entry; this is the subject of Proposition 2 below. For the time being, we investigate only what the best entry-deterring strategy is.

Now introduce some uncertainty $\tilde{\epsilon}$ in marginal cost and demand that is small in the sense that for all realizations ϵ of $\tilde{\epsilon}$, $c(\epsilon)$ lies in some small neighborhood of c^* and, for all x, $p(x,\epsilon)$ lies in a small neighborhood of $p^*(x)$. Assume too that

$$Ec(\tilde{\epsilon}) = c^* \quad \text{and} \quad Ep(x,\tilde{\epsilon}) = p^*(x) \quad \text{for all} \quad x\,. \tag{4}$$

Note that the second equation in (4) implies that

$$E\frac{dp}{dx}(x,\tilde{\epsilon}) = \frac{dp^*(x)}{dx} \quad \text{for all} \quad x\,.$$

Formulas (1), (2) and (4) together imply that

$$E[p(k_d^* + k_o, \tilde{\epsilon}) - q - c(\tilde{\epsilon})] = 0 \tag{5}$$

and

$$E[p(k_d^* + k_o, \tilde{\epsilon}) + k_d^* \frac{dp}{dk}(k_d^* + k_o, \tilde{\epsilon}) - c(\tilde{\epsilon})] \geq 0\,. \tag{6}$$

Now, if (6) holds with strict inequality, then, for small enough uncertainty,

$$p(k_d^* + k_o, \epsilon) + k_d^* \frac{dp}{dk}(k_d^* + k_o, \epsilon) - c(\epsilon) \tag{7}$$

is nonnegative for all realizations ϵ. This means that the incumbent will always produce at capacity, if it has installed k_d^* and the entrant k_o. That is, for all ϵ, the solution to

$$x(\epsilon) = \arg \max_{x \leq k_d^*} [p(x + k_o, \epsilon)x - c(\epsilon)x]$$

is

$$x(\epsilon) = k_d^*\,.$$

Suppose therefore that (6) holds with equality. Then, for some realizations ϵ, (7) must be negative, implying that the incumbent will produce below capacity. That is

$$x(\epsilon) < k_d^* \tag{8}$$

Because $p(\cdot, \epsilon)$ is decreasing, we have

$$p(x(\epsilon) + k_o, \epsilon) > p(k_d^* + k_o, \epsilon)$$

for such realizations, and hence

$$E[p(x(\tilde{\epsilon}) + k_o, \tilde{\epsilon}) - q - c(\tilde{\epsilon})] > 0\,.$$

We conclude that, with uncertainty, the incumbent must, in general, install capacity *greater* than k_d^* in order to deter entry. Now, even if (6) holds with equality, (2) and the concavity of $p^*(\cdot)$ imply that

$$p^*(k_d^*) + k_d^* \frac{dp^*}{dk}(k_d^*) - c^* > 0\,.$$

Therefore, for small enough uncertainty,

$$p(k_d^*, \epsilon) + k_d^* \frac{dp}{dk}(k_d^*, \epsilon) - c(\epsilon) > 0 ,$$

and so the incumbent always produces at capacity if the entrant is deterred. We conclude that the derivative of the incumbent's expected profit at k_d^* is

$$E\left[p(k_d^*, \tilde{\epsilon}) + k_d^* \frac{dp}{dk}(k_d^*, \tilde{\epsilon}) - c(\tilde{\epsilon}) - q\right] ,$$

which – because k_d^* exceeds monopoly capacity and $p(\cdot, \epsilon)$ is concave – is negative. Thus, the additional capacity reduces the incumbent's expected profit. We have established

Proposition 1. *The introduction of a small degree of uncertainty satisfying (4) either does not change or raises the optimal deterrence capacity of the incumbent. Moreover, if the degree of uncertainty is small, it either does not change or lowers the expected profit from deterring entry.*

The reader may be disturbed by the fact that we assumed small uncertainty, which implies that (6) must hold with equality, in order to infer a strictly increasing deterrence capacity. When there is a large degree of uncertainty, however, (7) can be negative even when (6) holds with strict inequality, and this is all we need to conclude that the deterrence capacity increases. (We invoke the small uncertainty assumption only because it simplifies the accommodation analysis below).

We now turn to the incumbent's behavior when it accommodates entry. In this case, its optimal capacity choice k_e^1 without uncertainty solves

$$\max_{k^1 \leq k_d^*} x_e^1(k^1)p^*(x_e^1(k^1) + x_e^2(k^1)) - c^*x^1(k^1) - qk^1$$

where, for $i = 1, 2$, $x_e^i(k^1)$ is firm i's equilibrium production in the subgame following the installation of capacity k^1 by firm 1 (assume that equilibrium in the subgame is unique). We claim that

$$k_e^1 = x_e^1(k_e^1) , \tag{9}$$

i.e., the incumbent installs no excess capacity in equilibrium. To see this, suppose, to the contrary, that

$$k_e^1 > x_e^1(k_e^1) .$$

Now if instead of k_e^1, the incumbent installs $\hat{k}^1 \equiv x^1(k_e^1)$ the entrant's equilibrium response $x_e^2(\hat{k}^1)$ cannot be greater than $x_e^2(k_e^1)$. (If it were greater, then, given that the best-response functions are negatively sloped, $x_e^1(\hat{k}^1)$ would be no greater than $x_e^1(k_e^1)$. But this means that $(x_e^1(\hat{k}_1), x_e^2(\hat{k}_1))$ would be an equilibrium in the subgame following k_e^1, a contradiction of uniqueness.) Hence, since best-response functions are negatively sloped, $x_e^1(\hat{k}^1) = \hat{k}^1$, and so the incumbent reaps a cost-saving of $q(k_e^1 - \hat{k}^1)$ by installing $\hat{k}^1$ instead of k_e^1. We conclude that (9) indeed must hold.

We next claim that

$$k^1 \frac{dp^*}{dk}(k^1 + x_e^2(k^1)) + p^*(k^1 + x_e^2(k^1)) > c^* \tag{10}$$

at $k^1 = k_e^1$. In view of (9), (10) implies that the constraint that the incumbent should be willing to produce at full capacity is not binding when $k^1 = k_e^1$. To see this, recall from (2) that the left-hand side of (10) is no less than the right-hand side when $k^1 = k_d^*$. Now, $k_d^* > k_e^1$. Therefore, it suffices to show that the left-hand side of (10) is decreasing in k^1. Implicitly differentiating the left-hand side of (10) with respect to k^1, we obtain

$$2\frac{dp^*}{dk} + k^1 \frac{d^2p^*}{d(k)^2} + \left(\frac{dp^*}{dk} + k^1 \frac{d^2p^*}{d(k)^2}\right) \frac{dx_e}{dk} . \tag{11}$$

By analogy with our analysis of $\frac{d\hat{x}^1}{dk}$, we can infer that $\frac{dx_e^2}{dk}$ is greater than -1. Hence, (11) is less than $\frac{dp^*}{dk}$, which is negative. Thus (10) holds after all.

Now, as before, introduce a small degree of uncertainty satisfying (4). Notice that the program

$$\max_{k^1 \le k_d^*} E\left[k^1 p(k^1 + k_e^2(k^1), \tilde{\epsilon}) - (c(\tilde{\epsilon}) + q)k^1\right] \tag{12}$$

such that

$$k_e^2(k^1) = \arg\max_{k^2} E\left[p(k^1 + k^2, \tilde{\epsilon})k^2 - (c(\tilde{\epsilon}) + q)k^2\right] \tag{13}$$

is linear in prices. Hence we can replace prices by their means and conclude that k_e^1 from above solves the program. Moreover, because (10) is a strict inequality,

$$k_e^1 \frac{dp}{dk}(k_e^1 + k_e^2, \epsilon) + p(k_e^1 + k_e^2(k_e^1), \epsilon) > c(\epsilon) , \tag{14}$$

for all realizations ϵ if uncertainty is small enough. But the fact that k_e^1 solves the program (12)–(13) and that (14) holds means that, with uncertainty, k_e^1 remains the optimal capacity choice for the incumbent if it allows entry. Hence, the incumbent's expected profit from allowing entry does not change with the introduction of uncertainty. However, Proposition 1 tells us that uncertainty can only *lower* the incumbent's profit from deterring entry. We can therefore state our main result:

Proposition 2. *The introduction of a small degree of uncertainty satisfying (4) either does not affect the incumbent's choice between deterrence and accommodation or else induces it to switch away from the former to the latter.*

In other words, uncertainty reduces the set of parameter values under which the incumbent will opt for deterrence.

4 A linear example

Let us set $p^* = 3$ and $q = c = 1$. Then, if there is no uncertainty, the deterrence capacity level k_d satisfies

$$(3 - k_o - k_d) - 2 = 0 .$$

That is, $k_d = 1 - k_o$ provided that $1 - k_o$ exceeds the monopoly capacity $\frac{1}{2}$, i.e., $k_o \leq \frac{1}{2}$, (notice that because

$$\frac{\partial}{\partial k}[(3 - k_o - k)k - k] = k_o \geq 0 \quad \text{at} \quad k = 1 - k_o ,$$

the incumbent will produce at full capacity should entry occur). Hence the incumbent's profit when it deters entry is

$$\Pi_d = (3 - 2 - (1 - k_o))(1 - k_o) = k_o(1 - k_o) . \tag{15}$$

If instead the incumbent accommodates entry and sets capacity k_1, the entrant responds by setting k_2 to maximize $(3 - k_1 - k_2) - 2k_2$, i.e.,

$$k_2 = \frac{1 - k_1}{2} .$$

Hence, the incumbent solves

$$\max_{k_1} \left[3 - k_1 - \left(\frac{1 - k_1}{2}\right)\right] - 2k_1 ,$$

i.e.,

$$k_1 = \frac{1}{2} .$$

And its profit with accommodation is thus

$$\Pi_a = \frac{1}{8} . \tag{16}$$

Hence, comparing (15) and (16), we conclude that deterrence occurs if and only if $\frac{1}{8} < k_o(1 - k_o)$. That is, the incumbent accommodates entry for

$$0 \leq k_o \leq \frac{2 - \sqrt{2}}{4} \tag{17}$$

and deters entry for

$$\frac{2 - \sqrt{2}}{4} \leq k_o \leq \frac{1}{2} . \tag{18}$$

Let us now introduce uncertainty about demand. Suppose that, with equal probabilities, $a = 3 + t$ (high demand) and $a = 3 - t$ (low demand), where $t = 2/3$. If the entrant has installed k_o units of capacity and demand turns out to be low, the incumbent faces the problem

$$\max \left(2\frac{1}{3} - k_o - k\right) k - k ,$$

and so will choose output

$$k = \frac{2}{3} - \frac{k_o}{2} .$$

But $\frac{2}{3} - \frac{k_o}{2} < 1-k$, if $k < \frac{2}{3}$. And so unlike in the case of certainty, the incumbent will not produce up to capacity when demand is low. This implies that, to deter entry, the incumbent must install capacity greater than $1 - k_o$. And so profit is less than $k_o(1 - k_o)$.

By contrast, if the incumbent accommodates entry, and if it and the entrant have installed capacities $\frac{1}{2}$ and $\frac{1}{4}$, respectively the incumbent chooses output to solve

$$\max_{k \leq \frac{1}{2}} \left(2\frac{2}{3} - \frac{1}{4} - k\right) k - k . \tag{19}$$

But the solution to (19) is $k = \frac{1}{2}$. That is, even when demand is low, the incumbent produces up to capacity. Hence, output and therefore expected profit are unchanged from the case of uncertainty. We conclude that $k_o = \frac{2-\sqrt{2}}{4}$ is no longer the cut-off point between accommodation and deterrence; k_o must be somewhat greater than $\frac{2-\sqrt{2}}{4}$ for deterrence to be worthwhile.

5 Discussion

The model we have analyzed treats the entrant and incumbent symmetrically from the standpoint of uncertainty. That is, each firm chooses capacity before uncertainty is resolved and output afterwards. Yet, it is unambiguously the incumbent who suffers from the uncertainty. The reason for this asymmetric effect is that the incumbent produces more than the entrant. Thus its marginal profit from production is lower, which means that it is less able than the entrant to commit itself to produce at capacity. To commit itself to produce at a certain level, a firm must ensure that its marginal profit there is nonnegative. This condition is less likely to be satisfied by the incumbent.

Differences in marginal profit also explain why a small degree of uncertainty reduces the incumbent's profit from deterring entry but not the profit from accommodating entry. In particular, the incumbent's output and total output are higher when the capacity choices are (k_d^*, k_o) than when they are $(k_e^1, k_e^2(k_e^1))$. Hence, its marginal profit is lower in the former case than in the latter (i.e., (6) may hold with equality, but (10) always holds with strict inequality). Thus, the incumbent has relatively greater trouble committing itself to producing at capacity when deterring rather than accommodating entry.

These arguments about marginal profit apply quite generally. We have, mainly for simplicity, modelled the post-entry game as Cournot quantity-setting. But precisely the same marginal considerations would enable us to deduce Propositions 1 and 2 with Bertrand price-setting. Indeed, any model in which the incumbent

(i) produces more than the entrant and (ii) produces more if it deters rather than accommodates entry is likely to lead to the same result.

To stress that uncertainty affects the incumbent and potential entrant quite differently, we deliberately modeled the timing of its resolution as symmetrically as possible with respect to the two firms. But we could readily consider other timings. For example, it might be natural to suppose that, at the point when the potential entrant decides on entry, much of the uncertainty about demand and costs has already been resolved. We could model this idea by assuming that the incumbent chooses capacity before and the entrant after uncertainty is realized. This alternative assumption would not affect our qualitative conclusions. If $k_d^1 = k_d^*$, then the introduction of a small degree of uncertainty implies that the entrant would find entry strictly profitable for favorable realizations. This means that, under uncertainty, the incumbent must install greater capacity to deter entry. However, the incumbent's expected profit is unaffected by uncertainty when it accommodates entry. Hence, as in Proposition 2, uncertainty reduces the incidence of entry-deterrence.

We have followed Schmalensee (1981) in supposing that the profitability of entry-deterrence derives from the minimum capacity of k_o that an entrant must install. Alternatively, we could have adopted Dixit's (1980) hypothesis that there is no minimum capacity but rather some other significant fixed cost of entry. It is easy to verify that this alternative specification makes no difference to our conclusions. Indeed, if we suppose, as in the previous paragraph, that the entrant can choose capacity *after* the resolution of uncertainty (but assume that the entry decision is made, and hence the fixed cost is incurred, before this resolution) then we reinforce our results by introducing, in addition, a conventional convexity effect, namely, the entrant's preference for uncertain demand and costs. If, under uncertainty, the entrant chooses the same level of output and capacity as under certainty, its expected profit will be the same as under certainty. Thus by adjusting capacity and output to the realization of demand and costs it can do strictly better, i.e., its profit is convex in demand and costs. Thus uncertainty makes entry more profitable and, hence, harder to deter. It was precisely this convexity that led Drèrze and Sheshinski (1976) to conclude that uncertainty increases entry in a competitive industry with set-up costs.

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