ON FIRST-BEST TAXATION

E. Maskin

Department of Economics, Massachusetts Institute of Technology

1. INTRODUCTION

The theoretical literature on the redistribution of income (or endowments) through taxation to achieve ethical goals has grown considerably in the past decade. Most of this work on optimal taxation attempts only second-best optimization. That is, authors generally begin with the supposition that the details about preferences and abilities necessary for making differential lump-sum transfers are lacking, so that only distortionary or poll taxes are available. In many cases, therefore, they abandon the possibility of attaining a full optimum.

Only rarely do authors state explicitly why this information should not be available.* Indeed, income tax returns could, in principle, be made sufficiently detailed to provide it. Probably the most important barrier to lump-sum transfers, as Hammond [1979] has persuasively argued, is that if consumers understand how the information they report about themselves is to be used, they will ordinarily have an incentive to report falsely.

This paper studies the possibility of first-best taxation when the tax authority is *ex ante* not completely informed about consumers' characteristics and so has to rely on consumers' reports. Following much of the literature on optimal taxation, I assume that tastes are publicly known, but that endowments (for example, labour skills) are private information. A tax schedule is first-best relative to a (Paretian) social welfare function (SWF) if it leads to the SWF being maximized subject only to feasibility constraints; i.e., if it leads to a full social optimum.

First-best taxes exist for some social welfare functions even when the government is not well-informed about consumers' characteristics, but precisely which social welfare functions can be fully optimized depends on the government's ability to observe consumers' actions. When consumers work, for example, the government may be able to observe their output or number of labour hours or both. If consumers vary in skill and skill itself is unobservable, one quantity may not be deducible from the other. When only one of the quantitities is observable, the consumer may, therefore, be able to *misrepresent* the size of his endowment. If the government is able to observe only output, for example, the consumer might claim to consume less (or more) leisure than, in fact, is the case, by pretending to be less (or more) skilled than he in fact is. Such misrepresentation could be worthwhile if taxes are a function of leisure (or labour).

In many circumstances, understatement of endowments is considerably easier than exaggeration. Suppose, for example, that the government can monitor hours worked perfectly but that it can observe only the output that consumers choose to show it.

* See, however, Dasgupta and Hammond (1978), and Mirrlees (1977).

Despite this imperfect monitoring, exaggeration of endowment is ruled out; a consumer cannot exhibit more output than he has produced. It is difficult to see, however, how to prohibit understatement so easily. Because of the greater plausibility of understatement than of exaggeration, I shall confine myself to that case. One type of understatement is *endowment concealment:* understatement of the truth by the consumer, where the unstated (concealed) portion is retained for his own consumption. A prominent practitioner of endowment concealment is the able worker who claims to be inept while continuing to work skilfully. Being able, he can complete tasks in less time than someone less skilled. If the tax scheme, however, is to compensate the less able for their longer work hours, he might well benefit from announcing to the world that he is a less skilled worker than he is, while continuing to enjoy the greater leisure of an able worker.

If, in this labour-production example, the government can observe both hours worked and output, a form of misrepresentation is still available to the consumer; namely, he can misrepresent the *quality* of his labour, by not only *claiming* to be, but by *behaving* as though he were less skilled than he actually is. By behaving in all ways as an inferior worker, the consumer is, in effect, *destroying* part of his endowment. Nonetheless, such destruction may be *advantageous* under many tax systems.

Concealment corresponds to what we commonly call *tax evasion* and is ordinarily ruled out by assumption in the tax literature. Indeed, the tax authority can prevent it by monitoring consumption. *Endowment destruction,* on the other hand, is more easily disguised. Destruction is probably the greatest theoretical barrier to a tax on skill.

Understandably, the class of social welfare functions that can be fully optimized is smaller when the consumer can only destroy his endowment than when he can conceal it. The optimizable class also typically depends on whether the economy is large or small. (A large economy is one where each consumer contributes negligibly to the aggregate endowment. I consider small economies in order to make the results applicable to such problems as division of goods among a small group of agents.) Roughly speaking, when concealment of endowments is permitted, only a competitive social welfare function (a social welfare function for which competitive allocations are the only first-best allocations) can be fully optimized in a large, private-goods economy, even when the distribution of endowments is known. In a small economy, not even this social welfare function is optimizable. On the other hand, in both large and small economies, a fairly extensive class, including the Rawlsian maximin social welfare function, can be optimized when individuals can only destroy their endowments. These results and others are presented in section 4 of this paper.

The implicit assumption in the preceding discussion, as well as in virtually all the optimum tax literature, is that a consumer's utility-maximizing behaviour, given the tax schedule he faces, is independent of the behaviour of others; he has a *dominant strategy*. This will certainly be the case if the consumer's taxes are dependent only on his own choices. Although tax schedules that make one consumer's payments dependent on the actions of another may seem rather complicated, there can be large gains to be made in using them (and thus dropping the requirement of dominant strategies), in terms of expanding the class of implementable social welfare functions. In section 5, therefore, I consider tax schemes where consumers, in effect, play a noncooperative game. I begin the formal part of the paper in section 2 with notation and terminology. Section 3 considers distribution of a single good while section 4 extends the analysis to two or more goods. Section 6 summarizes the main results in tabular form.

2. TERMINOLOGY AND NOTATION

I shall consider an economy with l private goods. Since I shall be concerned with distributional issues, I shall assume pure exchange, although virtually everything I say carries over to economies with production. When the economy is finite, let n be the number of consumers. In this case, an allocation is an n-dimensional vector, $(x^1, \ldots, x^n)$, with $x^i \in R^l_{++}$† for each i, where x^i is consumer i's consumption bundle. Allocation $\boldsymbol{x}=(x^1, \ldots, x^n)$ is aggregately feasible for initial endowments $\boldsymbol{\omega}=(\omega^1, \ldots, \omega^n)$ if $\Sigma_i x \leqslant \Sigma_i \omega^i$. Let U_∞ be the class of utility functions from R^l_{++} to R that are differentiable, monotonically increasing and concave and whose indifference curves do not intersect the axes. A social welfare function (SWF) is a mapping F: $R^n \times R^{nl}_{++} \to R$, with the interpretation that the allocation $(x^1, \ldots, x^n)$ is socially preferable to $(\bar{x}^1, \ldots, \bar{x}^n)$ for utility profile $(u^1, \ldots, u^n)$ and endowment profile $(\omega^1, \ldots, \omega^n)$ if and only if $F(u^1(x^1), \ldots, u^n(x^n); \omega^1, \ldots, \omega^n) > F(u^1(\bar{x}^1), \ldots, u^n(\bar{x}^n); \omega^1, \ldots \omega^n)$. Call F Paretian if, for all v, $v' \in R^n$, $v > v'$ implies $F(v) > F(v')$.

In the case of a continuum of consumers, suppose that i, indexing consumers, ranges in the interval [0,1]. An allocation is a (Borel-measurable) mapping $x[\cdot]$: $[0,1] \to R^l_+$ where $x[\text{i}]$ is consumer i's consumption bundle. The profile of initial endowments is given by the (Borel-measurable) mapping $\omega[\cdot]$: $[0,1] \to R^l_{++}$. Let Ω be the class of all such endowment profiles $\omega[\cdot]$. Feasibility entails that $\int x[\text{i}]d\text{i} = \int \omega[\text{i}]d\text{i}$. The profile of utility functions is given by a mapping $u[\text{i}]$: $[0,1] \to U$, where $u[\text{i}]$ is consumer i's utility function. Let $\mathscr{U}$ be the class of all utility profiles $u[\cdot]$. For a continuum of consumers, a social welfare functional is a mapping F: $I \times \Omega \to R$, where I is the space of (Borel-measurable) mappings from [0,1] to R, and Ω is the space of (Borel-measurable) mappings from [0,1] to R^l_{++}.

A tax scheme is a rule that associates a vector of taxes to each vector of messages provided by consumers. This paper considers messages which provide, among other things, some information, possibly untruthful, about the consumers' endowments. I shall suppose that consumer i's message space, S^i, is of the form $S^i = R^l_{++} \times M^i$, with typical element $s = (\omega^i, m^i)$. The ω^i component of the message is to be interpreted as consumer i's professed endowment, whereas m^i represents any additional information the consumer provides. As I mentioned in the introduction, I am particularly concerned with concealment and destruction of endowments. I therefore assume for each consumer i that if his true endowment is $\hat{\omega}^i$, then he may play those strategies with ω^i components satisfying $\omega^i \leqslant \hat{\omega}^i$. Formally, in the case of finitely many consumers, a tax scheme is a mapping T: $S^1 \times \ldots \times S^n \to R^{nl}$, such that if $\boldsymbol{s} = (s^1, \ldots s^n)$ is the vector of messages provided and $T(\boldsymbol{s}) = (T^1(\boldsymbol{s}), \ldots, T^n(\boldsymbol{s}))$, then $T^i(\boldsymbol{s})$ is consumer i's net transfer of commodities. For aggregate feasibility, I assume that $\Sigma_i T^i(\boldsymbol{s}) < 0$ for all $\boldsymbol{s}$. If consumer i can conceal his true endowment, $\hat{\omega}^i$, then his consumption bundle after taxes is $\hat{\omega}^i + T^i(\boldsymbol{s})$. If he is restricted to destruction only, and if $\boldsymbol{s} = ((\omega^1, m^1), \ldots, (\omega^i, m^i) \ldots. (\omega^n, m^n))$, then his final consumption is $\omega^i + T^i(\boldsymbol{s})$.

One can think of T as a game played by the consumers and, after adopting a suitable solution concept, one can speak of the equilibrium outcomes of T. Of course, nothing so far guarantees that outcomes be individually feasible, that consumers' final consumptions be non-negative. To avoid individual infeasibility I shall restrict attention to tax schemes with the property that, for each consumer i, for any $\boldsymbol{s}^{-i}$ (all components of the $\boldsymbol{s}$-vector except the i[th]) and any $s^i = (\omega^i, m^i)$, $\omega^i + T^i\ (s^i, \boldsymbol{s}^{-i}) \geqslant 0$. This assumption implies that, under either endowment concealment or destruction, all outcomes are feasible. It further implies that, under endowment concealment,

† R^l_{++} denotes the strictly positive orthant, whereas R^l_+ is the non-negative orthant.

consumer i must, in equilibrium, obtain at least the utility of his initial endowment, since if he takes $s^i=(0, m^i)$, he will, in fact, retain his endowment ('0' here is the zero vector).

For a given solution concept, let $E_T(\boldsymbol{u},\boldsymbol{\omega})$ be the set of equilibrium consumption allocation for tax scheme T when consumers are described by utility and endowment profiles $\boldsymbol{u}$ and $\boldsymbol{\omega}$. An outcome is a vector of final holdings. Let $G_F(\boldsymbol{u},\boldsymbol{\omega})=\{\boldsymbol{x}|\ F(u^1(x^1), \ldots, u^n(x^n);\ \omega)$ is maximized on the feasible set$\}$. I shall say that T is acceptable for the social welfare functional F (with respect to the given solution concept) for profile $\boldsymbol{u}$ if, for all profiles ω, $E_T(\boldsymbol{u},\omega) \subseteq F(\boldsymbol{u},\omega)$ and $E_T(\boldsymbol{u},\omega)\neq\phi$. That is, if T is imposed, an equilibrium allocation exists, and all equilibria are optimal with respect to F for any choice of endowments ω. The definition captures the idea that endowments are unknown to the game designer and that he must accept what the consumers tell him.

For any $\boldsymbol{u}$, F is implementable for $\boldsymbol{u}$ if for all ω and $\forall a \in G_F(\boldsymbol{u},\omega)$, there exists a tax scheme T, acceptable for F for $\boldsymbol{u}$, such that $a \in E_T(\boldsymbol{u},\omega)$. This definition conveys the idea that a social welfare function cannot properly be considered implementable unless all its optima are attainable through some tax scheme. A non-attainable optimum might just as well be looked on as not an optimum at all.

For some applications, I shall require the notion of differential implementability. For any $\boldsymbol{u}$, F is differentially implementable if, for each ω and $a \in G_F(\boldsymbol{u},\omega)$, an acceptable tax scheme T (as above) exists, with the additional provisos that $S^i=R^l_+$ for all i and that T is differentiable with respect to each consumer's strategy.

I am concerned in this paper with two non-cooperative solution concepts, those of dominant and Nash strategies. The optimal tax literature has, virtually without exception, restricted itself to dominant strategies. A consumer i with utility function u^i and endowment $\hat{\omega}^i$ has a dominant strategy $\bar{s}^i=(\bar{\omega}^i, \bar{m}^i)$ in tax scheme T if for all $s^i=(\omega^i,m^i)$ and all s^{-i}, $u^i(\hat{\omega}^i+T^i(\bar{s}^i,s^{-i}))\geq u^i(\hat{\omega}^i+T(s^i,s^{-i}))$ in the case of endowment concealment and $u^i(\bar{\omega}^i+T^i(\bar{s}^i,s^{-i}))\geq u^i(\omega^i+T^i(s,s^{-i}))$ for endowment destruction. That is, a strategy is dominant if the consumer is willing to use it regardless of the behaviour of others.

3. DOMINANT STRATEGIES: THE ONE-GOOD CASE

To begin the analysis, I shall consider the case $l=1$. This special case is of particular interest because many distributional questions involve the division of a single good, e.g. the distribution of income. There are several results. The first, entirely trivial, theorem, which applies to endowment concealment, states that, for given $\boldsymbol{u}$, the only social welfare functional that is implementable in dominant strategies for $\boldsymbol{u}$ is the 'no-transfer' SWF, i.e., the F such that $G_F(\boldsymbol{u},\omega)=\{\omega\}$. It holds for either small or large economies.

Theorem 3.1: For given $\boldsymbol{u}$, $F: R^n \times R^n_{++} \to R$ is implementable for u in dominant strategies when consumers can conceal their endowments if and only if $\forall\ \omega\ G_F(\boldsymbol{u},\omega)=\{\omega\}$ The same result holds for a continuum of consumers.

Proof of Theorem 3.1. For given $\boldsymbol{u}$, suppose that tax scheme T is acceptable for F for $\boldsymbol{u}$ (with respect to dominant strategies). By assumption on T, each consumer must in equilibrium get at least the utility of his initial endowment. Thus, in a one-good world he must, in fact, get his initial endowment. Thus, for any ω, $E_T(\boldsymbol{u},\omega)=\{\omega\}$, and so $G_F(\boldsymbol{u},\omega)=\{\omega\}$. The proof is the same for a continuum of consumers.

The second class of results characterizes the implementable SWF's for endowment destruction. Suppose first that the social welfare function F depends only on utilities (i.e. it is not a function of endowments). Then, theorem 3.2 asserts, essentially, that, if F is Paretian, it is implementable if and only if all utilities are 'normal goods' with respect to F. That is, if we think of F as a function representing preferences for different consumers' utilities, then, as 'income' rises, demand for each consumers' utility should also rise, if implementability is to be ensured.

Theorem 3.2‡: Suppose that $F: R^n \times R^n_{++} \to R$ is a continuous, twice piecewise differentiable function that is concave in its first n arguments. Suppose further that at any optimum all consumers have positive holdings. Consider the problem:

$$\max_{v^1, \ldots, v^n} F(v^1, \ldots, v^n; \omega^1, \ldots, \omega^n) \text{ subject to } \sum_i p_i v^i \leqslant y. \qquad (3.2.1)$$

If consumers are confined to endowment destruction, then F is implementable for all $\boldsymbol{u}$ if and only if, for each vector of positive prices $(p_1, \ldots, p_l)$ and for all i, the optimal v^i in the solution to (3.2.1) increases in y. The same result holds for a continuum of consumers.
Proof: See Appendix.

The intuition behind Theorem 3.2 is quite clear: an individual will be induced not to destroy his endowment if, by announcing a higher endowment, he obtains higher utility. If utility is a normal good, then raising income in the artificial maximization problem (3.2.1) will raise utility. If one draws an analogy between aggregate endowment and income (and the proof of theorem does this formally) the theorem follows.

The conditions on F under which all utilities are normal are messy to express; they consist of n ratios of determinants involving the first and second derivatives of F. Because of their messiness, I think it useful to state a simple special case of Theorem 3.2 which, nonetheless, probably covers most social welfare functions of interest; e.g. utilitarianism and the maximin criterion. Suppose that, in addition to being Paretian, a SWF F satisfies an equity property, a condition that prohibits discrimination against the well-endowed, and one additional condition on its second partial derivatives. Then it is implementable.

Theorem 3.3: Suppose that $F: R^n \times R^n_{++} \to R$ is a continuous, twice piecewise differentiable social welfare function that is concave in its first n arguments. Suppose further that, for all $i \neq j$

$$\partial^2 F/\partial v^i \partial v^j \geqslant 0, \quad \partial^2 F/\partial v^i \partial \omega^i \geqslant 0, \quad \text{and} \quad \partial^2 F/\partial v^i \partial \omega^j = 0, \qquad (3.2.2)$$

if these derivatives are defined. Then, if consumers are confined to endowment destruction, F is implementable in dominant strategies for all $\boldsymbol{u}$. The same result holds for a continuum of consumers.
Proof: The proof is simply a verification that, under the stated hypotheses, utilities are normal goods. The detailed argument is omitted.

The interpretation of the differential inequalities in the statement of Theorem 3.3 is quite straightforward. The condition $\partial^2 F/\partial v^i \partial v^j \geqslant 0$ is an equity condition. It states that one individual's improvement in welfare should not diminish another's weight in the social welfare function. The condition is satisfied by virtually all commonly discussed social welfare functions, including utilitarianism $(F(v^1, \ldots, v^n) = \Sigma v^i)$ and the Rawlsian maximin criterion $(F(v^1, \ldots, v^n) = \min v^i)$. It is sometimes argued that

‡ I am indebted to Kevin Roberts for a conversation during which a similar proposition emerged.

the maximin criterion is equitable or egalitarian, while utilitarianism is not. In terms of the behaviour of $\partial F/\partial v^i$ as v^j increases, this argument boils down to the observations that $\partial F/\partial v^i$ jumps from zero to one as v^j moves from below v^i to above v^i (in a two-person economy) for maximin social welfare, whereas $\partial^2 F/\partial v^i \partial v^j$ is identically zero under the utilitarian concept.

The condition $\partial^2 F/\partial v^i \partial \omega^i \geqslant 0$ prevents an individual's being penalized (in terms of his welfare weighting) solely because his endowment increases. I should emphasize that this condition is in no way anti-redistributive. In particular, the maximin criterion, which, assuming all utility functions are the same, prescribes equal consumption for all, satisfies it, since maximin makes *no* connection between one's endowment and one's welfare weighting. Indeed, any social welfare function (e.g. utilitarianism) in which weightings depend only on the distribution of utilities, rather than on the distribution of endowments, satisfies the condition $\partial^2 F/\partial v^i \partial \omega^i \geqslant 0$ automatically.

Corollary 3.4: The utilitarian ($F = \Sigma V_i$) and maximum ($F = \min V_i$) social welfare functions are implementable in dominant strategies if consumers are confined to endowment destruction.

I should emphasize that the sequences of quantifiers in Theorems 3.1 and 3.2 differ. The former theorem asserts that for any given profile of utility functions, F must take a particular form to be implementable. The latter (as well as Theorem 3.3) states that if F takes a certain form, then it is implementable for any $\boldsymbol{u}$.

4. DOMINANT STRATEGIES: TWO OR MORE GOODS

With two or more goods, the possibility of implementation for endowment concealment depends on whether the economy is large or small. I first characterize those SWF's that are implementable with a continuum of consumers under endowment concealment. The main result is that in a large economy a Paretian SWF is implementable if and only if it is competitive; i.e., optimal allocations are competitive allocations.

Theorem 4.1: Suppose that $F: I \times \Omega \to R$ is a Paretian social welfare function such that, for given $u[\cdot] \in \mathscr{U}_\infty$, for any given $\overline{\omega}[\cdot] \in \Omega$ and any $a \in G_F(u[\cdot], \overline{\omega}[\cdot])$, there exists a selection $V[\cdot]: \Omega \to \Omega$ with $V[\cdot](\omega[\cdot]) \in G_F(u[\cdot], \omega[\cdot])$ and $V[\cdot](\overline{\omega}[\cdot]) = a$ such that, for any consumer i, $V[\text{i}]$ is differentiable with respect to $\omega[\text{i}]$. For any such $V[\cdot]$, suppose that, if $\omega[\cdot], \omega'[\cdot] \in \Omega$ implies $\omega[\text{k}] = \omega'[\text{k}]$ for all $\text{k} \neq \text{i}$, then $V[\text{j}](\omega[\cdot]) = V[\text{j}](\omega'[\cdot])$ for all i and $\text{j} \neq \text{i}$. Then F is differentially implementable in dominant strategies if and only if F is the competitive SWF for $u[\cdot]$; i.e. $\forall\ \omega[\cdot] \in \Omega$ and $a \in G_F(u[\cdot], \omega[\cdot])$ if and only if a is a competitive equilibrium for $u[\cdot]$ and $\omega[\cdot]$.

Proof: See Appendix.

The hypotheses require, in effect, that, for any $a \in G_F(u[\cdot], \overline{\omega}[\cdot])$, we may choose a differentiable selection from $G_F(u[\cdot], \omega[\cdot])$ that includes a and that is differentiable with respect to each consumer's endowment. They imply, moreover, that any such selection must satisfy the property that changing a single consumer's endowment does not affect any other consumer's allocation.

Notice that Theorem 4.1 holds equally well if the tax authority knows the statistical distribution of endowments, provided that the distribution assigns positive density to any endowment which an individual might possibly profess to have. (If this last property were violated, then misrepresentation of endowments could be detected by observing that an individual's claim does not correspond to any endowment in the distribution). It therefore imposes a strong limitation on the possibility of first-best taxation. Indeed, if the economy is already competitive, no first-best taxes are possible.

The result is related to Theorem 5 in Hammond [1979]. The principal difference is that Hammond studies misrepresentation of *characteristics*, comprising both preferences and endowments, whereas I am concerned with the misrepresentation of endowments only. The distinction is significant because Hammond allows an individual to claim to have *any* characteristic, while I permit only *understatement* of endowment. One might have believed, *a priori*, that prohibition on overstatement would have allowed the implementation of SWF's other than the purely competitive one. (In the case of *Nash* implementation, the constraint on overstatement enormously increases the set of implementable SWF's); hence, the main motivation for Theorem 4.1. In my framework, Hammond's own theorem is about implementation possibilities when negative as well as positive concealment is allowed.

There are a number of other theorems, all of which establish competitive equilibrium as the unique optimum of certain SWF's or, often equivalently, as the unique equilibrium outcome of a certain game. The classical result along these lines is Edgeworth's observation, first formalized by Debreu and Scarf [1962], that, in a large economy, all core allocations are competitive. Recently, Mas-Colell [1978] has shown that, with a large number of players, the Nash equilibria of a trading game satisfying convexity, anonymity, non-degeneracy and neutrality properties must be competitive. Hurwicz [1979] has shown that, in the case where preferences are sufficiently rich, the only Paretian efficient and individually rational SWF's that are Nash implementable include the competitive allocations as optima.

Using Theorem 4.1, I can show that for small economies (strictly ones with only finitely many consumers) no Paretian social welfare function is implementable in dominant strategies when consumers can conceal their endowments. This is a result first proved by Postlewaite [1979]. Intuitively, the reason is that any implementable SWF for a finite economy has its infinite economy counterpart. But, as Theorem 4.1 shows, in an infinite economy, any implementable Paretian SWF in an infinite economy must be competitive. The same must be true in a finite economy. But it is then a simple matter to show that the competitive SWF is not implementable.

Theorem 4.2: If $F: R^n \times R^{nl}_{++} \to R$ is a Paretian social welfare function, then it is not differentially implementable in dominant strategies for any $\boldsymbol{u} \in U^n_\infty$ when consumers can conceal their endowments. Postlewaite, [1979].
Proof: See Appendix.

When consumers can misrepresent endowments only by destroying them, the possibilities for implementation improve. For given utility profile $\boldsymbol{u}$, the range of implementable SWF's is, in general, quite considerable. Nonetheless, the SWF's that are implementable for *all* profiles $\boldsymbol{u}$ are limited to the class of SWF's with 'L-shaped' social indifference curves.

Theorem 4.3: Suppose $F: R^n \times R^{nl}_{++} \to R$ is a Paretian, continuous, piecewise differentiable SWF. F is differentially implementable for all $\boldsymbol{u} \in U^n_\infty$ under endowment destruction, if and only if there exists an SWF, F^*, of the form:

$$F^* (\boldsymbol{u},\omega) = \max_t \{t \,|\, a^i (t,\omega) \leqslant u^i \text{ for all i}\}, \tag{4.3.1}$$

where, for all i, a^i is monotonically increasing in t and ω^i,

$\lim_{t\to\infty} a^i = \infty$ and $\lim_{t\to-\infty} a^i = -\infty$, such that $\forall \boldsymbol{u}\ \forall \boldsymbol{\omega}\ G_F(\boldsymbol{u},\boldsymbol{\omega}) = G_{F^*}(\boldsymbol{u},\boldsymbol{\omega})$.

Proof: See Appendix.

Condition (4.3.1) amounts to saying that F has 'L-shaped' social indifference contours. Various authors, including Dasgupta and Hammond [1978], Postlewaite [1979], Mirrlees [1977], have remarked that when $a_i(t,\omega) = a_i t$ (where a_i is a positive constant), F^* is implementable.

5. NASH STRATEGIES

The preceding sections demonstrate that, with dominant strategies, possibilities for first-best taxation are (at least with more than one good) quite limited. In this section, I weaken the requirement of dominant strategies by adopting Nash equilibrium as the solution concept. By making a consumer's tax depend not only on his own strategy but on those of others, I show that the class of implementable social welfare functions is very wide. Indeed, individual rationality—the property that ensures that each consumer obtains at least the utility of his initial endowment—is a necessary and sufficient condition for implementability when consumers can conceal their endowments. Still weaker conditions suffice for implementation when consumers are confined to destruction.

Definition: A social welfare function F is *individually rational* if $\forall i\ \forall \boldsymbol{u}\ \forall \boldsymbol{\omega}\ \forall \boldsymbol{x} \in G_F(\boldsymbol{u},\boldsymbol{\omega})$, $u^i(x^i) \geqslant u^i(\omega^i)$

Theorem 5.1: Let $F\colon R^n_{++} \times R^{nl}_{++} \to R$ be a social welfare function. For $\boldsymbol{u}$, F is implementable for $\boldsymbol{u}$ in Nash strategies if and only if F is individually rational.

Proof: It is clear that an implementable F must be individually rational because a consumer could always conceal (and, hence, consume) his whole endowment. To see that individual rationality implies implementability, *see* the constructive proof in Hurwicz, Maskin and Postlewaite [1979].

Definition: A SWF, F, is *non-confiscatory* if $\forall i\ \forall \boldsymbol{\omega}\ \forall \boldsymbol{u}\ \forall \boldsymbol{x} \in G_F(\boldsymbol{u},\boldsymbol{\omega})$, $x^i > 0$.

Theorem 5.2: A non-confiscatory SWF is implementable in Nash strategies for all $\boldsymbol{u}$, if consumers are confined to endowment destruction.

Proof: See Hurwicz, Maskin and Postlewaite [1979].

6. SUMMARY

It may be useful to summarize the main results of this paper in two tables.

Table 1. Paretian SWF's implementable in dominant strategies.

	Concealment	Destruction
Finite number of consumers:		
Economies with only one good	No transfer SWF	'Normal' SWF's
Economies with 2 or more goods	None	SWF's with 'L-shaped' indifference curves
Continuum:		
Economies with only one good	No transfer SWF	'Normal' SWF's
Economies with 2 or more goods	Competitive SWF	SWF's with 'L-shaped' indifference curves

Table 2. SWF's implementable in Nash strategies.

Concealment	Individually Rational SWF's
Destruction	Non-confiscatory SWF's

APPENDIX

In this appendix, I provide the proofs of those results (excluding Theorem 5.1 and 5.2) that were not proved in the main text.

Proof of Theorem 3.2: Suppose that $F: R^n \times R^n_{++} \to R$ satisfies the hypotheses of the theorem. Consider any $\bar{u},\bar{\omega}$ and $x \in G_F(\bar{u},\bar{\omega})$. For the time being assume that F is twice differentiable, instead of just twice piecewise differentiable. Then, because optima are interior, we can choose $\bar{x}^*$: $R^n_+ \to R^n$ such that $\bar{x}^*(\cdot)$ is differentiable for all $\omega, \bar{x}^*(\omega) \in G_F(\bar{u},\omega)$ and $\bar{x}^*(\bar{\omega}) = \bar{x}$. That there exists a tax scheme T, acceptable for F for u, such that $\bar{x} \in E_T(u,\omega)$, is equivalent to the property

$$\forall\omega \ \frac{\partial x^{i*}}{\partial \omega^i}(\omega) > 0. \tag{A.1}$$

(because, if equation (A.1) holds, an individual cannot gain by understating his endowment and, therefore, telling the truth is a dominant strategy).

Consider problem (3.2.1) theorem 3.2. For given $\mathring{\omega}$, and any i let

$$p_i = \frac{\partial F}{\partial v_i}(\bar{u}_1(x^{1*}(\mathring{\omega}), \dots, \bar{u}_n(x^{n*}(\mathring{\omega})); \omega) \tag{A.2}$$

and

$$y = \max\left\{\Sigma p_i v_i \mid \forall i \quad v_i = \bar{u}_i(x^i), \quad \sum_i x^i = \sum_i \omega^i\right\}. \tag{A.3}$$

Thinking of y as a function of ω, we can infer that $\partial y/\partial\omega^i > 0$. Let $V_l^{1*}(y), \dots, V_l^{n*}(y)$ be a (differentiable) solution to (3.2.1) with p_i and y as defined in (A.2) and (A.3). Then, for all i,

$$\frac{\partial \bar{u}^i}{\partial x}\frac{\partial x^{i*}}{\partial \omega^i} = \frac{\partial V^i}{\partial y}\frac{\partial y}{\partial \omega^i}.$$

Since $\partial\bar{u}^i/\partial x$ is positive, we conclude that $\partial x^{i*}/\partial\omega^i$ is positive if and only if $\partial v^i/\partial y$ is positive. That is, F is implementable only if all utilities are normal goods. If F is only piecewise differentiable, the same argument can be applied to each point in a sequence of closer and closer differentiable approximations to complete the proof.

Proof of Theorem 4.1: Suppose, first, that F is differentially implementable for $u[\cdot]$. Then $\forall\, \omega[\cdot]\ \forall\, a \in G_F(u[\cdot], \omega[\cdot])$ there exists an acceptable tax scheme $T[\cdot]$ where $S^i = R^l$; where $a \in E_T(u[\cdot], \omega[\cdot])$ and where $T[i]$ is differentiable with respect to consumer i's strategy. Without loss of generality (*see* Dasgupta, Hammond and Maskin [1979]), we may assume, that professing the truth is a dominant strategy. Because F is Paretian, we may invoke the second fundamental welfare theorem to conclude that there exist $p(\omega[\cdot]) \in R^l_+$ and $Y[\cdot](\omega[\cdot]) \in I$ with

$\int_{j\in[0,1]} Y[j](\omega[\cdot])dj = 0$, such that $t^i = T[i](\omega[\cdot])$ solves $\max_{t^i} u^i(\omega^i + t^i)$ such that

$p(\omega[\cdot]) \cdot t^i \leq Y[i](\omega[\cdot])$. That is, the equilibrium outcome is a competitive allocation with transfer payments (the $Y[i]$'s are the transfers).

Choose $\omega[\cdot] \in \Omega$. For any i and any $\omega^i \in R^l_+$, define $W_{\omega^i}[\cdot] \in \Omega$ so that

$$W_{\omega}[j] = \begin{cases} \overline{\omega}[j], & j \neq i \\ \omega^i, & j = i \end{cases}$$

Because $T[\cdot]$ $(\omega[\cdot])$ is a selection from $G_F(u[\cdot],\omega[\cdot])$, the hypotheses imply that $\forall j \neq i$ so $T[j]$ $(\overline{\omega}[\cdot]) = T[j]$ $(W_{\omega^i}[\cdot])$. Therefore $p(\overline{\omega}[\cdot]) = p(W_{\omega^i}[\cdot])$. Thus, for all i,$\omega^i = \overline{\omega}^i$ maximizes $u[i](\omega^i + T[i](W_{\omega^i}[\cdot]))$ subject to $\omega^i < \overline{\omega}^i$. From the Kuhn-Tucker theorem, there exist, for each consumer i, γ_k (k=1, ..., l) such that

$$\sum_{j=1}^{l} \frac{\partial u[i]}{\partial x_j} \frac{\partial T_j[i]}{\partial \omega_k[i]} = \gamma_k$$

at the maximum. Also, at the maximum, $p(\omega[\cdot]) \cdot T[i](W_{\omega^i}[\cdot]) = Y[i](W_{\omega^i}[\cdot])$, so that $\Sigma_j p_j(\partial T_j[i]/\partial \omega_k[i]) = \partial Y[i]/\partial \omega_k[i]$. Therefore, since p is proportional to $(\partial u[i]/\partial x_1, \ldots, \partial u[i]/\partial x_l)$ at the maximum (equilibrium is interior because $u[\cdot] \in \mathscr{U}_\infty$), $\partial Y[i]/\partial \omega_k[i] \geqslant 0$ for each k. That is, the income transfer that consumer i receives must be a non-decreasing function of the endowment he professes. Now suppose that, for some i, $Y[i](\overline{\omega}[\cdot]) < 0$. Then choose $\mathring{\omega}^i \leqslant \overline{\omega}[i]$ such that $((\partial u[i]/\partial x_1)(\mathring{\omega}^i), \ldots, (\partial u[i]/\partial x_l)(\mathring{\omega}^i))$ is proportional to $p(\mathring{\omega}[\cdot])$. (Such a choice is possible because $u[i] \in \mathscr{U}_\infty$).

Now if $Y[i](W_{\mathring{\omega}^i}[\cdot]) < 0$, then

$\max \{u[i](\mathring{\omega}^i + t^i) | p(\overline{\omega}[\cdot]) \cdot t^i \leqslant Y[i]\,(W_{\mathring{\omega}^i}[\cdot])\} < u[i]\,(\mathring{\omega}^i)$.

But, if consumer i's endowment is $\mathring{\omega}^i$, he can guarantee himself the utility $u[i](\mathring{\omega}^i)$ by concealing his endowment. Thus $Y[i](W_{\mathring{\omega}^i}[\cdot]) \geqslant 0$.

But this, in combination with $Y[i](\overline{\omega}[\cdot])$, contradicts the fact that $Y[i]$ must be increasing in ω^i. Thus, for all i, $Y[i](\overline{\omega}[\cdot]) \geqslant 0$.

Since $\int Y[i](\overline{\omega}[\cdot])di = 0$ (transfers sum to zero), we conclude that $Y[i](\overline{\omega}[\cdot]) = 0$ for (almost) all i. Thus, the equilibrium outcome is a competitive equilibrium. We have shown, therefore, that any implementable SWF must be competitive. To see the converse, choose $\overline{\omega}[\cdot]$ $a \in G_F(u[\cdot], \omega[\cdot])$ and a selection $V(\omega[\cdot])$ from $G_F(u[\cdot], \omega[\cdot]$ such that $a \in V[\cdot](\omega[\cdot])$ as above. Define $T[\cdot](\omega[\cdot]) = V[\cdot](\omega[\cdot])$. It is straightforward to verify that T is acceptable for F.

Proof of Theorem 4.2: Suppose that F: $R^n \times R^{nl}_{++} \to R$ satisfies the hypotheses of the theorem but is implementable for some $\bar{\boldsymbol{u}} \in U^n_\infty$. Choose $u[\cdot] \in \mathscr{U}_\infty$ such that, $\forall i \in [0,1]$, $\bar{u}[i] = \bar{u}[S(in)]$, where $S(x)$ is the smallest integer greater than or equal to x. Consider F^*: $I \times \Omega \to R$ such that $\forall\ \omega[\cdot]$, G_{F^*} $(\bar{u}[\cdot], \omega[\cdot])$ is defined as follows:

(i) If there exists $\overline{\boldsymbol{\omega}}$ such that, for each $k \in \{1, \ldots, n\}$, $\omega[i/n] = \overline{\omega}^k$ for almost all i with $(k-1) \leqslant i \leqslant k$, then $x[\cdot] \in G_{F^*}$ $(\bar{U}[\cdot], \omega[\cdot])$ if and only if (a) there exists $\overline{\mathbf{x}} \in G_F(\bar{\boldsymbol{u}}, \overline{\boldsymbol{\omega}})$ such that $\forall i$ $x[i/n] = \bar{x}^i$ if $\omega[i/n] = \overline{\omega}^{s(i)}$ and (b) if $\omega[i/n] \neq \omega^{s(i)}$ for some i, then there exists $\overline{\overline{x}} \in G_F(\bar{\boldsymbol{u}}, (\omega^{s(i)}, \boldsymbol{\omega}^{-s(i)}))$ such that $x[i/n] = \overline{\overline{x}}^{-s(i)}$.

(ii) If no such $\overline{\boldsymbol{\omega}}$ exists, then $x[\cdot] \in G_{F^*}(\bar{u}[\cdot], \omega[\cdot])$ if and only if $x[\cdot]$ is a competitive equilibrium with respect to $\bar{u}[\cdot]$ and $\omega[\cdot]$.

Intuitively, F^* is a replication of F if the continuum economy itself is a replication of an n-person economy, (i.e., if (i) applies) and the competitive SWF otherwise, (i.e., if (ii) applies). It is easy to see that F^* is differentially implementable: if (i) applies then consumers tell the truth for the same reason they do when F is the SWF; if (ii) applies, then consumers tell the truth, because, from theorem 4.1, the competitive SWF is implementable. But, from theorem 4.1 again, F^* itself must be the competitive SWF, since it is implementable. Because F^* is just a replication of F, we conclude that

F must be the competitive SWF. Therefore it remains only to show that the competitive SWF is not implementable for any choice of $\boldsymbol{u} \in U^n_\infty$. To do this, let us assume, for simplicity, that $n=2$ and $l=2$ (the argument is essentially the same for larger n and l). Choose $\bar{\boldsymbol{u}} \in u^2_\infty$. If the competitive social welfare function is differentially implementable for $\boldsymbol{u}$, then there exists an acceptable game form $T: R^l_{++} \times R^l_{++} \to R^l \times R^l$, differentiable in each argument, such that, for all $\omega=(\omega^1, \omega^2)$, $(\omega^1+T^1(\omega^1, \omega^2)$, $\omega^2+T^2(\omega^1, \omega^2))$ is a competitive equilibrium allocation for $\bar{\boldsymbol{u}}$ and ω. Let $p(\omega)$ be the prices associated with the equilibrium allocation $(\omega^1+T^1(\omega^1, \omega^2), \omega^2+T^2(\omega^1, \omega^2))$. By the assumptions on $\bar{\boldsymbol{u}}$ and T, p is differentiable with respect to ω^1 and ω^2. Now if consumer 1 is to be induced not to conceal any of his endowment, then for all ω we must have

$$\frac{\partial u^1}{\partial x_1}\left(\omega^1 + T^1(\omega)\frac{\partial T^1_1}{\partial \omega^1_k}(\omega) + \frac{\partial u^1}{\partial x_2}(\omega^1 + T^1(\omega))\right)\frac{\partial T^1_2}{\partial \omega^1_k}(\omega) \geqslant 0 \text{ for k} = 1,2. \quad \text{(A.4)}$$

Furthermore, from Walras' Law

$$p_1\frac{\partial T^1_1}{\partial \omega^1_k} + p_2\frac{\partial T^1_2}{\partial \omega^1_k} + T_1\frac{\partial p_1}{\partial \omega^1_k} + T_2\frac{\partial p_2}{\partial \omega^1_k} = 0, \text{ for k}=1,2. \quad \text{(A.5)}$$

Hence, from (A.4) and (A.5)

$$T^1_1\frac{\partial p_1}{\partial \omega^1_k} + T^1_2\frac{\partial p_2}{\partial \omega^1_k} \leqslant 0. \quad \text{(A.6)}$$

At every point in consumer l's consumption space, at least one of the two goods is (locally) normal for him.

Without loss of generality, suppose that good 1 is normal in a neighbourhood of radius ε of $\bar{\omega}^1$. Choose $\bar{\omega}^2$ for consumer 2 so that (A.4) $\omega^{-1}+T^1(\boldsymbol{\omega}^{-1})$ is within ε of ω^{-1}, (A.5) $\bar{\omega}+T(\bar{\omega})$ lies on the convex portions of the two consumers offer curves and (A.6) $T^1_2(\bar{\omega})<0$. (Since the offer curves must be convex in a neighbourhood of the endowment, conditions (A.4) and (A.5) can be met simply by choosing $\bar{\omega}^2$ so that the resulting equilibrium does not lie too far from the initial endowments.)

Let $x^1(p_1,p_2,\omega^1)$ be consumer l's utility-maximizing consumption bundle, given prices (p_1,p_2) and endowment ω^1. Then

$$\frac{\partial}{\partial \omega_2}(\omega^1_2 - x^1_2) = \frac{p_1}{p_2}\frac{\partial x^1_1}{\partial \omega_2} > 0,$$

since good 1 is normal. That is, an increase in the endowment of good 2 induces consumer 1 to sell more of it; his offer curve falls. Because consumer 2's offer curve is convex in a neighborhood of $\bar{\omega}+T(\bar{\omega})$, the fall in the offer curve means that p_2 falls relative to p_1; i.e., if prices are normalized to lie in the unit simplex $\partial p_2/\partial \omega^1_2<0$ and $\partial p_1/\partial \omega^n_2>0$. Because $T^1_2(\bar{\omega})<0$, and $T^1_1(\bar{\omega})>0$, we have, therefore,

$$T^1_1\frac{\partial p_1}{\partial \omega^1_k} + T^1_2\frac{\partial p_2}{\partial \omega^1_k} > 0, \quad \text{(A.7)}$$

which contradicts equation (A.6). Thus F is not implementable after all.

Proof of theorem 4.3: Suppose that an F^* as hypothesized in equation (4.3.1) exists. For given endowments $\boldsymbol{\omega}$, F can be maximized by maximizing the following Lagrangian:

$$\max\ t+\sum_i \lambda^i(u^i(x^i)-a^i(t,\omega))+\sum_j \gamma_j(\sum_i x^i_j-\sum_i \omega^i_j). \tag{A.8}$$

The first order conditions for an (interior) maximum are

$$1=\sum_i \lambda^i \frac{\partial a^i}{\partial t}=0. \tag{A.9a}$$

$$\lambda^i \frac{\partial u^i}{\partial x^i_k}+\gamma_j=0 \quad \text{for all i, j} \tag{A.9b}$$

$$u^i=a^i \quad \text{for all i} \tag{A.9c}$$

$$\sum_i x^i=\sum_i \omega^i \tag{A.9d}$$

Let $\{x^{i*}\}$ and $\{t^*\}$ be a solution to the problem. Thinking of x^{i*} and t^* as functions of ω and making use of (A.9c) we obtain

$$\sum_k \frac{\partial u^i}{\partial x^i_k}\frac{\partial x^{i*}_k}{\partial \omega^i_j}=\frac{\partial a^i}{\partial t}\frac{\partial t^*}{\partial \omega^i_j}+\frac{\partial a^i}{\partial \omega^i_j} \quad \text{for all i, j.} \tag{A.10}$$

From (A.9b), if $\gamma_j>0$ for some j, then $\lambda^i<0$ for all i. But this contradicts (A.9a), since $\partial a^i/\partial t\geqslant 0$. Therefore, $\gamma_j\leqslant 0$ for all j. Now $\partial t^*/\partial\omega^i_j=-\gamma_j$. Therefore, from $\partial a^i/\partial\omega^i_j\geqslant 0$ and (A.10), we conclude that

$$\sum_k \frac{\partial u^i}{\partial x^i_k}\frac{\partial x^{i*}_k}{\partial \omega^i_j}\geqslant 0 \quad \text{for all i and j,}$$

implying that consumers will never understate their endowments. F is implementable.

Next suppose that F^* as hypothesized does not exist. Then not all F's indifference curves are 'L-shaped'. In particular, there exists a vector $\bar{v}\in R^n$ and integers i and j such that $(\partial F/\partial v_i)(\bar{v})\neq 0\neq(\partial F/\partial v_j)(\bar{v})$. For simplicity suppose $n=l=2$. In that case, if one of F's indifference curves is not L-shaped, there exists $\bar{v}$ such that $(\partial F/\partial v_1)(\bar{v})\neq 0\neq(\partial F/\partial v_2)(\bar{v})$.

Choose $\boldsymbol{u}\in U^2_\infty$ and $x\in R^4_{++}$ such that

$$(u^1(\bar{x}^1),\ u^2(\bar{x}^2))=\bar{v} \quad \text{and} \tag{A.11}$$

$$\frac{\partial F}{\partial v_1}\frac{\partial u^1}{\partial x_1}=\frac{\partial F}{\partial v_2}\frac{\partial u^2}{\partial x_1} \quad\text{and}\quad \frac{\partial F}{\partial v_1}\frac{\partial u^1}{\partial x_2}=\frac{\partial F}{\partial v_2}\frac{\partial u^2}{\partial x_2} \quad \text{at } \boldsymbol{x}=\bar{\boldsymbol{x}}. \tag{A.12}$$

Thus, $\bar{x}\in G_F(\boldsymbol{u},\boldsymbol{x})$. Differentiating (A.12) with respect to ω_1 and supposing that $\partial u^1/\partial x_1$ and $\partial u^1/\partial x_2$ are nearly zero, we obtain

$$\frac{\partial x^{1\prime}_1}{\partial\omega_1}=\frac{\begin{vmatrix} \frac{\partial F}{\partial v_2}\left(\frac{\partial^2 u^2}{\partial x_1^2}+\frac{\partial^2 u^2}{\partial x_1\partial x_2}\right) & \frac{\partial F}{\partial v_1}\frac{\partial^2 u^1}{\partial x_1\partial x_2}+\frac{\partial F}{\partial v_2}\frac{\partial^2 u^2}{\partial x_1\partial x_2} \\ \frac{\partial F}{\partial v_2}\left(\frac{\partial^2 u^2}{\partial x_1\partial x_2}+\frac{\partial^2 u^2}{\partial x_2^2}\right) & \frac{\partial F}{\partial v_1}\frac{\partial^2 u^1}{\partial x_2^2}+\frac{\partial F}{\partial v_2}\frac{\partial^2 u^2}{\partial x_2^2} \end{vmatrix}}{D} \tag{A.13a}$$

and

$$\frac{\partial x^{1}_{2}}{\partial \omega_1} = \frac{\begin{vmatrix} \dfrac{\partial F}{\partial v_1}\dfrac{\partial^2 u^1}{\partial x_1^2} + \dfrac{\partial F}{\partial v_2}\dfrac{\partial^2 u^2}{\partial x_1^2} & \dfrac{\partial F}{\partial v_2}\left(\dfrac{\partial^2 u^2}{\partial x_1^2} + \dfrac{\partial^2 u^2}{\partial x_1 \partial x_2}\right) \\ \dfrac{\partial F}{\partial v_1}\dfrac{\partial^2 u^1}{\partial x_1 \partial x_2} + \dfrac{\partial F}{\partial v_2}\dfrac{\partial^2 u^2}{\partial x_1 \partial x_2} & \dfrac{\partial F}{\partial v_2}\left(\dfrac{\partial^2 u^2}{\partial x_1 \partial x_2} + \dfrac{\partial^2 u^2}{\partial x_2^2}\right) \end{vmatrix}}{\mathrm{D}} \tag{A.13b}$$

where

$$\mathrm{D} = \begin{vmatrix} \dfrac{\partial F}{\partial v_1}\dfrac{\partial^2 u^1}{\partial x_1^2} + \dfrac{\partial F}{\partial v_2}\dfrac{\partial^2 u^2}{\partial x_1^2} & \dfrac{\partial F}{\partial v_1}\dfrac{\partial^2 u^1}{\partial x_1 \partial x_2} + \dfrac{\partial F}{\partial v_2}\dfrac{\partial^2 u^2}{\partial x_1 \partial x_2} \\ \dfrac{\partial F}{\partial v_1}\dfrac{\partial^2 u^1}{\partial x_1 \partial x_2} + \dfrac{\partial F}{\partial v_2}\dfrac{\partial^2 u^2}{\partial x_1 \partial x_2} & \dfrac{\partial F}{\partial v_1}\dfrac{\partial^2 u^1}{\partial x_2^2} + \dfrac{\partial F}{\partial v_2}\dfrac{\partial^2 u^2}{\partial x_2^2} \end{vmatrix}. \tag{A.13c}$$

Suppose that u^1 and u^2, in addition to satisfying (A.11) and (A.12), satisfy

$$\begin{pmatrix} \dfrac{\partial F}{\partial v_1}\dfrac{\partial^2 u^1}{\partial x_1^2} & \dfrac{\partial F}{\partial v_1}\dfrac{\partial^2 u^1}{\partial x_1 \partial x_2} \\ \dfrac{\partial F}{\partial v_1}\dfrac{\partial^2 u^1}{\partial x_1 \partial x_2} & \dfrac{\partial F}{\partial v_1}\dfrac{\partial^2 u}{\partial x_1^2} \end{pmatrix} = \begin{pmatrix} -1 & 4 \\ 4 & -17 \end{pmatrix}$$

and

$$\begin{pmatrix} \dfrac{\partial F}{\partial v_2}\dfrac{\partial^2 u^2}{\partial x_1^2} & \dfrac{\partial F}{\partial v_2}\dfrac{\partial^2 u^2}{\partial x_1 \partial x_2} \\ \dfrac{\partial F}{\partial v_2}\dfrac{\partial^2 u^2}{\partial x_1 \partial x_2} & \dfrac{\partial F}{\partial v_2^2}\dfrac{\partial^2 u^2}{\partial x_2^2} \end{pmatrix} = \begin{pmatrix} -2 & 3 \\ 3 & -5 \end{pmatrix} \tag{A.14}$$

Then, from (A.13*a*) and (A.13*b*),

$$\frac{\partial x^1}{\partial \omega_1} = \frac{-8}{17} \text{ and } \frac{\partial x^2}{\partial \omega_1} = \frac{-1}{17}.$$

Thus

$$\frac{\partial u^1}{\partial \omega_1}(x({}^2\omega)) = \frac{\partial u^1}{\partial x_1}\frac{\partial x^1}{\partial \omega_1} + \frac{\partial u^1}{\partial x_2}\frac{\partial x^2}{\partial \omega_1} < 0.$$

Therefore, individual 1 has the incentive to understate his endowment, so that F is not implementable after all.

ACKNOWLEDGEMENTS

I am grateful to Peter Hammond for a long and very useful written discussion of my paper. I wish to thank him, James Mirrlees and Kevin Roberts for helpful conversations. The paper owes much to the work of Peter Hammond, Leo Hurwicz, James

Mirrlees, and Andrew Postlewaite. Indeed, theorem 4·2 of the theorems in the section on Nash equilibrium is derived from some on-going joint work by Hurwicz, Postlewaite, and me. I would like to thank the NSF for financial support.

REFERENCES

Dasgupta P. and Hammond P. J. (1978) *Fully Progressive Taxation*, Univ. Essex.

Debreu G. and Scarf H. (1963) A Limit Theorem on the Core of an Economy. *Int. Econom. Rev.*

Hammond P. J. (1979) Straightforward Individual Incentive Compatibility in Large Economies. *Rev. Economic Studies.*

Hurwicz L. (1972) On Informationally Decentralized Systems. In McGuire C. and Radner R. (ed.) *Decision and Organization*, Amsterdam, North-Holland.

Hurwicz L. (1979) On Allocations Attainable through Nash Equilibria. In Laffont J.-J. (ed.) *Aggregation and Revelation of Preferences*, Amsterdam, North-Holland.

Hurwicz L., Maskin E. and Postlewaite A. (1979) *Feasible Implementation of Social Choice Correspondences by Nash Equilibria*. Massachusetts Inst. Technology.

Mas-Colell A. (1978) *An Axiomatic Approach to the Efficiency of Non-cooperative Equilibrium in Economics with a Continuum of Traders*. IMSSS Technical Report No. 274.

Mirrlees J. (1977) *The Theory of Optimal Taxation*. Univ. Oxford.

Postlewaite A. (1979) Manipulation via Endowments. *Rev. Economic Studies.*

DISCUSSION

by **P. Hammond,** *University of Essex, and Stanford University*

Introduction: The Theoretical Limits to Redistribution

In my view it is entirely appropriate that the first paper of this symposium should be an examination of the theoretical limits to redistribution. And Eric Maskin's paper, which is part of his work on the intriguing subject of 'incentive compatibility', goes about exploring these limits in entirely the right sort of way, as I see it. It helps to bridge an important gap in the approaches which economists have become accustomed to using, as I shall now try to explain.

On the one hand, competent second-year economics undergraduates learn about lump-sum taxes and subsidies as means of redistributing income. Lump-sum taxes are often rather carelessly defined; they are taxes where the amount collected from any one agent is fixed in nominal terms—in particular, they are independent of the agent's market transactions. Thus, there is no divergence between prices before tax and prices after tax. Loosely speaking, there are no 'distortions' in the economy. In particular, by comparison with the income taxes which are often thought of as the right instruments for redistributing income, there is no disincentive to work, because the implicit marginal rate of tax is zero if the tax is a lump-sum tax. So, with lump-sum taxes there are no limits at all to redistribution.

The trouble with lump-sum taxes, as we all know, and as Samuelson ([1947], 247–8) and Graaff ([1957], 77–79) at least have been careful to point out, is that one wants to base these taxes on individual characteristics such as ability which are hard to observe directly. And if one does base a tax on ability, say, or the quality of labour actually provided, this creates disincentives regarding the *quality* rather than the quantity of labour. Individuals become reluctant to acquire skills or to reveal them by taking skilled jobs. (For an analysis of an optimal ability tax, in Mirrlees' 1971 model of income taxation, see Dasgupta and Hammond [1978]).

So one has lump-sum taxes on one side of the gap. They represent an impracticable ideal, perhaps, but at least their theoretical implications are well understood. Competent third-year economics undergraduates, at least if they specialize in public finance, then make a considerable leap to consider the effects and effectiveness of commodity taxes, income taxes, property taxes, etc. which are based very closely on market transactions. Distortions are introduced and quantities become affected. Again, the theory is fairly well developed, especially since the work of Diamond and Mirrlees [1971] on optimal commodity taxation.

The question which should be asked, however, before we resort to income taxes, commodity taxes, etc. is whether these really represent the best we can do. If we rule out lump-sum taxes because even they create undesirable incentive effects, need we go over all the way to the other, more customary, taxes of public finance which also create undesirable incentive effects? In some cases, and with some major qualifications, it seems that we must (as I have shown elsewhere in Hammond [1979]), but Maskin shows that in other cases a considerable degree of compromise may be possible. In the present context, although generally it may not be possible to have full redistribution of income, it may still be possible to have much more redistribution than recent public finance literature suggests.

By now it should be fairly clear that to bridge the gap between lump-sum taxes and public finance we need to consider rather carefully the problem of how to devise a mechanism for redistributing real income in the economy, or, more generally, for allocating resources. We should be careful to understand exactly what kind of information the mechanism relies on, and what incentives individuals have for providing that information. These are precisely the issues to which the work on incentive compatibility, including Maskin's paper, addresses itself.

In fact, the title, 'On First-Best Taxation', is itself quite thought-provoking. Usually, 'first-best' welfare economics assumes that lump-sum taxation is used to bring about an ideal income distribution; 'first-best taxation', therefore, should be optimal lump-sum taxation. But, as I have already argued, the lump-sum taxes one would like to have may rely on information about skills and other individual characteristics which individuals will choose to suppress, so many schemes of lump-sum taxation are simply not feasible. Taking into account the information which individuals choose not to suppress, Maskin's tax schemes really are 'first-best'.

For the most part, Maskin considers a general exchange economy of l private goods and n consumers. In the present paper, it is assumed that all consumers' preferences are known although elsewhere, in his work with Hurwicz and Postlewaite, Maskin considers what happens when they are not. It is endowments which are unknown. Production and public goods can also be included. The problem is to devise a scheme of taxation—a 'mechanism'—which yields good outcomes even when individuals choose to conceal or even to destroy some of their endowments. It is, I think, rightly assumed that individuals cannot overstate their endowments, because such overstatement can usually be detected.

Broadly speaking, there are two classes of mechanism which can be distinguished, both of which are considered. The first and simplest kind is a 'direct' mechanism in which each individual simply reports an endowment (not necessarily his true endowment) and nothing else. The second kind is an 'indirect' mechanism which, in the present paper, relies on each individual sending not only a report of his endowment but also an extra message in addition—very often a report of other agents' endowments. Where the mechanism is direct, dominant strategies are of especial interest, as considered in sections 3 and 4 of the paper. With indirect mechanisms, one looks instead for Nash strategies; section 5 summarizes the relevant results from Maskin's work with Hurwicz and Postlewaite.

The Simplest Case: Redistribution in a One-Good Economy

In a one-good economy, all that can be done is to redistribute the individuals' initial endowments of the single good, which will be called 'cake'. A transfer scheme, or mechanism, redistributes cake in response to individuals' messages. In section 3 of the paper, these messages are direct reports of endowments.

Now, in section 2, Maskin requires each individual to be able to achieve a non-negative consumption level whatever he reports. So, if he is allowed to conceal his endowment, the individual can always ensure that none of it is taken away from him by reporting a zero endowment. Thus an equilibrium outcome of the mechanism can never be worse for the individual than if he had just kept his original endowment—in other words, the outcome is 'individually rational'.

In a one-good economy of course, individual rationality means that, since nobody can finish up with less of the one good than he started off with, so nobody can finish up with more, and thus we stay where we started, with the initial endowment. This is rather uninteresting, (*see* Theorem 3.1).

Where endowments cannot simply be concealed, however, but have to be destroyed, then equilibrium outcomes need not be individually rational, and so interesting possibilities arise even in a one-good economy. Then formally, under endowment destruction, with $T^i(\omega)$ denoting the direct transfer mechanism, and $x^i(\omega)$ denoting i's resulting consumption, we have:

$$x^i(\omega) = T^i(\omega) + \omega^i \quad (\text{if } \omega^i \leqslant \overline{\omega}^i).$$

Provided that $dT^i/d\omega^i > -1$, which ensures that $dx^i/d\omega^i > 0$, then obviously i's dominant strategy is to announce $\overline{\omega}^i$.

This possibility is of some considerable interest. It includes, for instance, the perfect redistribution rule (under inequality aversion):

$$T^i(\omega) = \frac{1}{n} \sum_{j=1}^{n} \omega^j - \omega^i$$

under which, when every individual tells the truth,

$$x^i(\omega) = \frac{1}{n} \sum_{j=1}^{n} \omega^i$$

because, of course, $dT^i/d\omega^i = (1/n) - 1$. In fact, any 'income' redistribution rule which maximizes a sum of (strictly concave) utility functions:

$$W = \sum_{i=1}^{n} u^i(x^i) \text{ subject to } \sum_{i=1}^{n} x^i = \sum_{i=1}^{n} \omega^i$$

can be truthfully implemented in this way, provided that $du^i/dx^i > 0$ and $d^2u^i/d(x^i)^2 < 0$ for every i and every $x^i \geqq 0$. In this one-good economy then, when unreported endowments are destroyed, there are no effective limits to redistribution for a large class of social welfare functions, including utilitarianism and maximin. Maskin's Theorems 3.2 and 3.3 give far more general social welfare functions which are implementable in this way.

So much for direct mechanisms in the one-good economy. Let me now turn to indirect mechanisms, as in Maskin's section 5 on Nash equilibrium. When endowments can be concealed, Theorem 5.1 is not so interesting in a single good economy because, as I have already remarked, only the trivial mechanism of transferring nothing at all is individually rational in such an economy. Where endowments cannot be concealed, however, Theorem 5.2 is striking. It is based on constructing a very specific mechanism where each consumer announces an entire profile of endowments, but has to destroy his own endowment if he understates it. The mechanism is remarkably powerful, even in a one-good economy. It can implement any transfer rule under which, if the total endowment is positive, each agent always get some of it. The mechanism works by allocating all the total endowment to a single consumer unless all consumers announce identical endowment profiles. It encourages each consumer to claim that he has a large endowment himself. Other agents are encouraged not to understate one another's endowments, but to be the lowest overstaters. If just one agent overstates another's endowment, however, that overstater get nothing. By these extreme reallocations out of equilibrium, each consumer, as I say, is induced to state the true profile of endowments. There is no need to assume individual rationality or even (surprisingly enough) the monotonicity property which is usually needed for Nash implementation (Dasgupta, Hammond and Maskin [1979], Theorem 7.1.3).

Before moving on to a general many-good economy, let me remark on one special case which can be reduced to a one-good economy, even though it has many goods. This is an economy where equilibrium price ratios are fixed and known, either because the economy is a small open one with all goods traded, or because the nonsubstitution theorem applies (i.e. there are constant returns to scale, no joint production, and a single non-produced factor). Some combination of these two cases will also suffice, of course. Then the efficiency prices which support any Pareto-efficient allocation are effectively exogenous, and so all that matters is the value of each consumer's endowment at these prices. Of course, this is really just a version of Hicks' composite commodity theorem.

Redistribution with Many Goods: Direct Mechanisms

When there are many goods, the economy faces the problem of achieving an efficient allocation, as well as of trying to redistribute income. Leaving consumers with their initial endowments is most unlikely to be Pareto-efficient. But then, once one tries to move to towards the Pareto frontier, consumers will typically be able to gain by concealing their true endowments, as Maskin's Theorem 4.2 shows. Notice that any Pareto-efficient allocation is a competitive equilibrium with suitable lump-sum transfers, provided preferences are convex and the allocation is an interior one. So, in the pure exchange economy of Theorem 4.2, one can think of a price and transfer mechanism for producing efficient allocations. Now, if prices are held fixed, lump-sum transfers are likely to cause difficulties, as they did in the one-good economy (unless unreported endowments are destroyed). But if lump-sum transfers are ruled out, and prices are varied, we are using a competitive or Walrasian mechanism without lump-sum transfers. Then consumers expect to benefit by claiming to have smaller endowments of goods they are selling, which tends to raise those prices relative to the prices of the goods they are buying. That is true, at least, in a gross substitutes economy. And it provides a very loose and heuristic argument for the impossibility of achieving Pareto efficiency—by dominant strategies, at least—and goes a little way towards explaining Theorem 4.2.

When unreported endowments are destroyed, however, the situation does change somewhat. Note, however, that the competitive mechanism may well be vulnerable to destruction of endowments. We all know that a coffee-producing country may want to dump some of its coffee at sea, in a glut year, in order to drive up the price sufficiently to increase its total revenue. And a classroom example may serve to reinforce the point rather dramatically.

Suppose we have an economy with two goods—left gloves and right gloves—and two agents—one with an endowment of left gloves and one with an endowment of right gloves. Both agents want equal numbers of left gloves and right gloves; excess gloves of either kind, however, are worthless. In such an economy, unless the total endowments of left and right gloves happen to be exactly equal, one kind of glove must have a zero price. For suppose there are more right gloves than left gloves, and that all gloves have a positive price. Then the total demands for left gloves and right gloves must be equal, so both markets cannot clear simultaneously. Thus right gloves become a free good, and the agent who started with an endowment of right gloves never gets any left gloves, while the lucky agent who started off with left gloves gets the right gloves needed to match. This is the competitive equilibrium, which is highly inequitable. Suppose, however, that the original owner of the right gloves destroys some of his endowment, so that it is now left gloves which predominate. Then the equilibrium switches around; left gloves have a zero price, and their original owner gets no pairs of gloves, while the original owner of right gloves now gets all the pairs it is possible to put together. Obviously, then, each agent wants to destroy some of his endowment. This is true even if initially the number of left and right gloves are equal.

Theorem 4.3, however, shows that there are mechanisms, other than the competitive mechanism, in which agents will not want to destroy their endowments. Postlewaite [1979] has already presented an example of such a mechanism. Maskin presents a general class. In fact, the allocation mechanism, in a pure exchange economy, must be the one which maximizes a function of the form:

$$\min_{i} \varphi^i(u^i(x^i);\ \omega)$$

i.e. each consumer's utility gain from trade, measured in some interpersonally comparable utility units. It is required that φ^i be increasing in u^i but, when any component of ω^i increases, then φ^i does not rise relative to φ^j (all $j \neq i$). Then an optimal allocation has the property that each consumer does gain from having a larger endowment. So no consumer wants to understate his endowment. By making sure that consumers are actually expected to provide more if their endowments are larger, it is also possible, as in a one-good economy, to arrange that no consumer wants to overstate his endowment either.

Redistribution with Many Goods: Profile Mechanisms

When there are many goods, and when agents can keep their endowments whether they declare them or not, there is little one can achieve in small economies with direct mechanisms as Maskin's Theorem 4.2, discussed previously, clearly demonstrates. It is in these circumstances that the ingenious Maskin mechanisms exhibited elsewhere in the proofs of Theorems 5.1 and 5.2 come into their own.

In constructing these mechanism, Maskin has followed an idea due originally, I understand, to Karl Vind. Each agent, instead of reporting just his own characteristic, as he would with a direct mechanism, reports an entire profile of characteristics. In fact, perhaps we should call these 'profile' mechanisms.

The profile mechanism exhibited in the proof of Theorem 5.2 works, it should be recalled, when agents cannot conceal their own endowments. Also, it is expressly designed to deal with a many-good economy; when I described its operation in a one-good economy, that was merely for simplification. The extension to many goods is actually rather straightforward.

When agents are free to conceal their own endowments then, as has been argued above, for a social choice rule to be implementable at all by a profile mechanism, it must be individually rational. Theorem 5.1 claims that any individually rational social welfare function is implementable in Nash strategies.

The proofs of Theorems 5.1 and 5.2 both rest on the construction of particular profile mechanisms. Some of the features of this kind of profile mechanism are not entirely satisfactory. In the mechanism used to prove Theorem 5.1, all agents are expected to report not only an entire common profile of endowments, but also a common allocation which is optimal given those endowments. Now, if it is the true profile of endowments which is to be reported then it is reasonably simple for the agents to coordinate their reported profiles. But there may be alternative Nash equilibria in which the true profile is not reported, and some agents may be better off in an alternative Nash equilibrium. More seriously, the agents need to agree, in effect, on an allocation, and for this there is no obvious allocation to fall back on (as they can fall back on the true profile) unless it happens that the optimum is unique. In fact, we know that there must be cases where the optimum is *not* unique, because typically a social choice rule which always selects a unique optimum cannot be implemented (see Dasgupta, Hammond and Maskin [1979], Theorem 7.2.3). Where the optimum is not unique, of course, different agents will try to get different allocations put forward for the common proposal, and it is not at all clear how a Nash equilibrium could ever be arrived at.

Labour Endowments and Redistribution of Income

Sections 3 and 4 of Maskin's paper show how direct mechanisms could implement a certain class of allocation rules in an economy provided that consumers could not over-report their endowments and provided that under-reported endowments were destroyed. I agree with Maskin that these assumptions can make good sense when it is skills which are not known. A worker reveals his skill by exercising it in the job he does; with suitable monitoring of job performance, the worker who does not reveal his skill destroys it, in effect, because of course his pay relates to the skills he does reveal. On the other hand, with monitoring, a worker cannot pretend to have skills he actually lacks, and in that sense he cannot over-report his skills.

This can be demonstrated somewhat more formally. Let x denote a consumption vector, l denote hours worked, and q a vector of skills the worker uses in his job. Each worker, i, we assume, has a known utility function $u^i(x^i, l^i, q^i)$ (with q^i as an argument because workers are not indifferent to the type of job they do). In addition, worker i has a true vector of skill $\bar{q}^i$ and can supply any skill vector $\bar{q}^i$ satisfying the vector inequality $q^i \leqq \bar{q}^i$.

In this economy, a direct mechanism specifies each worker's consumption vector $x^i(\overline{\boldsymbol{q}})$, as well as hours worked $l^i(\overline{\boldsymbol{q}})$ and skills required $q^i(\overline{\boldsymbol{q}})$, as functions of the profile of skill announcements $\overline{\boldsymbol{q}}$. Feasibility depends upon how skill reacts with labour supply in the production set; in general, we can say that:

$$\sum_i x^i(q) \in X(\boldsymbol{l},\boldsymbol{q})$$

where $X(\boldsymbol{l},\boldsymbol{q})$ is the set of aggregate outputs of consumption goods which are possible given the profile of hours worked $\boldsymbol{l}$ and skills used $\boldsymbol{q}$. Implementability rests on the mechanism $\boldsymbol{x}(\bar{\boldsymbol{q}}),\boldsymbol{l}(\bar{\boldsymbol{q}}),\boldsymbol{q}(\bar{\boldsymbol{q}})$ rewarding skill—in the sense that:

$$q^i \geqq q^i \text{ implies } u^i(x^i(q^i,\bar{\boldsymbol{q}}^{-i}),l^i(q^i,\bar{\boldsymbol{q}}^{-i});\bar{q}^i) \geqslant u^i(x^i(\bar{\boldsymbol{q}}),l^i(\bar{\boldsymbol{q}});\bar{q}^i)$$

and also on skills being fully exploited—in the sense that $q^i(\bar{\boldsymbol{q}})=\bar{q}^i$ (all i, $\bar{\boldsymbol{q}}$).§ The first condition ensures that no agent wants to underreport his skill, while the second ensures that exaggerated reports will be caught, because the worker will be asked to provide skills he does not have if he does exaggerate. On the assumption that more skill is productive and that doing a more skilled job provides no less utility (if x and l are unaffected) then any allocation which does not use all the skills of the workers in the economy is Pareto dominated by another which does. So 'good' allocation rules will then use all available skills. The problem then is simply to choose the best rule which does reward skill. Very often, it seems, the maximin rule will be the best rule of this kind (cf. Dasgupta and Hammond [1978]).

Conclusions

Maskin's paper is a useful and provocative exploration of the theoretical limits to the redistribution of incomes arising from the need to maintain incentives. His indirect profile mechanisms are powerful ways of implementing what are technically called 'individually rational' allocation rules. However, the requirement for each agent to report an entire profile of endowments in such mechanisms limits their plausibility somewhat. On the other hand, the superficially more appealing direct mechanisms do not work at all well in small economies except in special cases where unreported endowments are destroyed and over-reported endowments are not allowed (or else get corrected). Yet the case many of us have in mind in this connection, where it is labour skills which are unequally distributed and income which we wish to redistribute, is a special case of this kind. Then it is possible to use taxes on skill to get much closer to equality of utility, if not of income, than has previously been recognized.

All the analysis is based on the assumption that consumers' tastes are known. In particular, this implies that we know who is lazy and who is industrious. With tastes unknown, implementation becomes much less easy. This is an interesting topic for future research. The general conclusion survives, however: the theoretical limits to redistribution may be much less severe than is commonly thought, and in particular, income taxation is a very poor substitute for skill taxation when hours of work are flexible.

REFERENCES

Dasgupta P. S. and Hammond P.J. (1978) Fully Progressive Taxation, University of Essex Economics Discussion Paper No. 115. *J. Pub. Econ.* (forthcoming).

Dasgupta P. S., Hammond P. J. and Maskin E. S. (1979) The Implementation of Social Choice Rules: Some General Results on Incentive Compatibility. *Rev. Economic Studies* **46,** 185–216.

Diamond P. A. and Mirrlees J. A. (1971) Optimal Taxation and Public Production. *Am. Econom. Rev.* **61,** 8–27 and 261–78.

Graaff J. de V. (1957) *Theoretical Welfare Economics.* Cambridge Univ.

Hammond P. J. (1979) Straightforward Individual Incentive Compatibility in Large Economies. *Rev. Economic Studies* **46,** 263–82

Postlewaite A. (1979) Manipulation Via Endowments. *Rev. Economic Studies* **46,** 255–62.

Samuelson P. A. (1947) *Foundations of Economic Analysis.* Harvard Univ.

§ The expression $\bar{\boldsymbol{q}}^{-i}$ here signifies the profile $\bar{\boldsymbol{q}}$ without the component $\bar{q}^i$.

SUMMARY OF THE GENERAL DISCUSSION

Collard asked whether it might be possible for people to avoid disclosing their true endowment to the tax authorities while still managing to gain the advantages of revealing their full skills to their employer. One might argue to the tax authorities that one's good job was due to luck, over-valuation, etc., and that one's basic skill was not responsible for one's high position. Hammond agreed that it was an important assumption that skills could be observed, albeit indirectly, and that the tax authorities had access to the same information as employers in this respect.

Cowell wondered whether the assumption that unrevealed endowments would be lost was really a good one, particularly in the case of labour endowments. For instance, one way of avoiding the screening of endowments would be for people to become self-employed, in which case it would become much harder for the tax authorities to determine the quantity and quality of their labour inputs. Maskin agreed that this possibility was an important constraint on the nature of the optimal tax system. In his model, consumers always had the option of consuming their endowment instead of entering into trade, and this might be interpreted as self-employment. In order to encourage consumers to enter the market, the tax system must leave them with greater benefits than the non-market option.

Mirrlees said that Maskin's results were very interesting, but he would have liked to hear more about some of the questions raised, particularly the difficulties associated with Nash equilibria. There would be in principle many Nash equilibria and there would be a problem of ranking and choosing the best one, through the SWF. At the end of section 4 Maskin lists a number of cases where his mechanisms work, but exactly how restricted must the form of the SWF be to assure implementability? Maskin replied that the extent to which the SWF must be restricted depended on the potential possibilities about endowments and tastes in the economy. If we only made the general assumptions that preferences were convex, monotonic and continuous, then a dictatorial SWF would be required. But if we could assume Cobb–Douglas utility functions, uniqueness would be assured anyway. He felt that the more serious objection to Nash equilibria was the problem of how such equilibria might arise. There was no adequate theory of the dynamics by which a Nash equilibrium might be reached from an initial disequilibrium. One of the attractions of the dominance concept was that reaching equilibrium with dominant strategies was far more plausible.

Hammond asked whether a consistent allocation might be arrived at even where the individual agents were not telling the truth. Maskin replied that in Nash equilibrium it would never be optimal to lie, although one individual might perceive that he could be better off outside a particular Nash equilibrium by lying. Cowell wondered whether the results of the paper might suggest that the government should be happy with a situation in which individuals were generally understating their tax liabilities if such a situation seemed to work. Maskin agreed. The aim was not to make people to tell the truth, but to get the best possible allocation of resources.

Collard sought to draw out the implications of the paper for the theme of the conference. A practical method of first-best taxation was really the philosophers' stone of redistributionists; a mechanism for making truthful declaration a dominant strategy would undoubtedly be ideal, but seemed to be quite impossible. It was more possible to engineer a game with Nash equilibria, but this concept had difficulties. More work might provide some more answers, but meanwhile one must soldier on with second-best solutions.

LeGrand wondered whether the paper could be summed up as 'the optimal amount of tax evasion'. Maskin disagreed. He felt that his optimum described a position in which evasion did not occur. LeGrand said that was true of the dominant solution but not of the non-dominant case. Kay said that when the aim was to tax an unobservable attribute, there was a difference between the policy of finding some other attribute that is observable and taxing that, and the policy of taxing the unobservable and taking the consequences of misreporting and evasion.

Hammond felt that the real lesson of the paper was that we might be able to get closer to lump-sum taxation than we had previously thought.